Level 4

PRECALCULUS READINESS DIAGNOSTIC TEST PRACTICE

Directions: Study the samples, work the problems, then check your answers at the end of each topic. If you don't get the answer given, check your work and look for mistakes. If you have trouble, ask a math teach or someone else who understands the topic.

TOPIC 1: ELEMENTARY OPERATIONS with NUMERICAL and ALGEBRAIC FRACTIONS

A. Simplifying fractions (by reducing):

example: $\frac{27}{36}=\frac{9\bullet 3}{9\bullet 4}=\frac{9}{9}\bullet\frac{3}{4}=1\bullet\frac{3}{4}=\frac{3}{4}$

(note that you must be able to find a common factor–in this case 9–in both the top and bottom in order to reduce a fraction).

example: $\frac{3a}{12ab}=\frac{3a\bullet 1}{3a\bullet 4b}=\frac{3a}{3a}\bullet\frac{1}{4b}=1\bullet\frac{1}{4b}=\frac{1}{4b}$

(common factor: $3a$)

Problems 1-8: Reduce:

1. $\frac{13}{52}=$
2. $\frac{26}{65}=$
3. $\frac{3+6}{3+9}=$
4. $\frac{6axy}{15by}=$
5. $\frac{14x+7y}{7y}=$
6. $\frac{5a+b}{5a+c}=$
7. $\frac{x-3}{3-x}=$
8. $\frac{4(x+2)(x-3)}{(x-3)(x-2)}=$

B. Equivalent fractions (equivalent ratios):

example: $\frac{3}{4}$ is equivalent to how many eighths?

$\frac{3}{4}=\frac{}{8},\ \frac{3}{4}=1\bullet\frac{3}{4}=\frac{2}{2}\bullet\frac{3}{4}=\frac{2\bullet 3}{2\bullet 4}=\frac{6}{8}$

example: $\frac{6}{5a}=\frac{}{5ab},\ \frac{6}{5a}=\frac{b}{b}\bullet\frac{6}{5a}=\frac{6b}{5ab}$

example: $\frac{3x+2}{x+1}=\frac{}{4(x+1)},\ \frac{3x+2}{x+1}=\frac{4}{4}\bullet\frac{3x+2}{x+1}=\frac{12x+8}{4x+4}$

example: $\frac{x-1}{x+1}=\frac{}{(x+1)(x-2)}$,

$\frac{x-1}{x+1}=\frac{(x-2)(x-1)}{(x-2)(x+1)}=\frac{x^2-3x+2}{(x+1)(x-2)}$

Problems 9-13: Complete:

9. $\frac{4}{9}=\frac{}{72}$
10. $\frac{3x}{7}=\frac{}{7y}$
11. $\frac{x+3}{x+2}=\frac{}{(x-1)(x+2)}$
12. $\frac{30-15a}{15-15b}=\frac{}{(1+b)(1-b)}$
13. $\frac{x-6}{6-x}=\frac{}{-2}$

C. Finding the lowest common denominator:

(LCD) by finding the least common multiple (LCM) of all denominators:

example: $\frac{5}{6}$ and $\frac{8}{15}$. First find LCM of 6 and 15:

$6=2\bullet 3$

$15=3\bullet 5$

$\text{LCM}=2\bullet 3\bullet 5=30$,

so $\frac{5}{6}=\frac{25}{30}$, and $\frac{8}{15}=\frac{16}{30}$

example: $\frac{3}{4}$ and $\frac{1}{6a}$:

$4=2\bullet 2$

$6a=2\bullet 3\bullet a$

$\text{LCM}=2\bullet 2\bullet 3\bullet a=12a$,

so $\frac{3}{4}=\frac{9a}{12a}$, and $\frac{1}{6a}=\frac{2}{12a}$

example: $\frac{2}{3(x+2)}$ and $\frac{ax}{6(x+1)}$:

$3(x+2)=3\bullet(x+2)$

$6(x+1)=2\bullet 3\bullet(x+1)$

$\text{LCM}=2\bullet 3\bullet(x+1)\bullet(x+2)$,

so $\frac{2}{3(x+2)}=\frac{2\bullet 2(x+1)}{2\bullet 3(x+1)(x+2)}=\frac{4(x+1)}{6(x+1)(x+2)}$,

and $\frac{ax}{6(x+1)}=\frac{ax(x+2)}{6(x+1)(x+2)}$

Problems 14-18: Find equivalent fractions with the lowest common denominator:

14. $\frac{2}{3}$ and $\frac{2}{9}$
15. $\frac{3}{x}$ and 5
16. $\frac{x}{3}$ and $\frac{-4}{x+1}$
17. $\frac{3}{x-2}$ and $\frac{4}{2-x}$
18. $\frac{x}{15(x^2-2)}$ and $\frac{7x(y-1)}{10(x-1)}$

D. Adding and subtracting fractions:

If denominators are the same, combine the numerators:

example: $\frac{3x}{y}-\frac{x}{y}=\frac{3x-x}{y}=\frac{2x}{y}$

If denominators are different, find equivalent fractions with common denominators:

example: $\frac{a}{2}-\frac{a}{4}=\frac{2a}{4}-\frac{a}{4}=\frac{2a-a}{4}=\frac{a}{4}$

example: $\frac{3}{x-1}+\frac{1}{x+2}=\frac{3(x+2)}{(x-1)(x+2)}+\frac{(x-1)}{(x-1)(x+2)}$

$=\frac{3x+6+x-1}{(x-1)(x+2)}=\frac{4x+5}{(x-1)(x+2)}$

Problems 19-26: Find the sum or difference as indicated (reduce if possible):

19. $\frac{4}{7}+\frac{2}{7}=$
20. $\frac{3}{x-3}-\frac{x}{x-3}=$
21. $\frac{b-a}{b+a}-\frac{a-b}{b+a}=$
22. $\frac{x}{x-1}+\frac{x}{1-x}=$
23. $\frac{3}{a}-\frac{1}{2a}=$
24. $\frac{3x-2}{x-2}-\frac{2}{x+2}=$
25. $\frac{2x-1}{x+1}-\frac{2x-1}{x-2}=$
26. $\frac{1}{(x-1)(x-2)}+\frac{1}{(x-2)(x-3)}-\frac{2}{(x-3)(x-1)}=$

E. Multiplying fractions:
Multiply the top numbers, multiply the bottom numbers, reduce if possible:

example: $\frac{3}{4} \bullet \frac{2}{5} = \frac{6}{20} = \frac{3}{10}$

example: $\frac{3(x+1)}{x-2} \bullet \frac{x^2-4}{x^2-1} = \frac{3(x+1)(x+2)(x-2)}{(x-2)(x+1)(x-1)} = \frac{3x+6}{x-1}$

27. $\frac{2}{7a} \bullet \frac{ab}{12} =$

28. $\frac{3(x+4)}{5y} \bullet \frac{5y^3}{x^2-16} =$

29. $\frac{(a+b)^3}{(x-y)^2} \bullet \frac{(x-y)}{(5-p)} \bullet \frac{(5-p)^2}{(a+b)^2} =$

F. Dividing fractions:
Make a compound fraction and then multiply the top and bottom (of the larger fraction) by the LCD of both:

example: $\frac{a}{b} \div \frac{c}{d} = \frac{\frac{a}{b}}{\frac{c}{d}} = \frac{\frac{a}{b} \bullet bd}{\frac{c}{d} \bullet bd} = \frac{ad}{bc}$

example: $\frac{7}{\frac{2}{3} - \frac{1}{2}} = \frac{7 \bullet 6}{\left(\frac{2}{3} - \frac{1}{2}\right) \bullet 6} = \frac{42}{4-3} = \frac{42}{1} = 42$

example: $\frac{5x}{2y} \div 2x = \frac{\frac{5x}{2y}}{2x} = \frac{\frac{5x}{2y} \bullet 2y}{2x \bullet 2y} = \frac{5x}{4xy} = \frac{5}{4y}$

30. $\frac{3}{a} \div \frac{b}{3} =$

31. $\frac{\frac{x+7}{x^2-9}}{\frac{1}{x-3}} =$

32. $\frac{a-4}{\frac{3}{a} - 2} =$

33. $\frac{2a-b}{\frac{1}{2}} =$

Answers:

1. $\frac{1}{4}$
2. $\frac{2}{5}$
3. $\frac{3}{4}$
4. $\frac{2ax}{5b}$
5. $\frac{2x+y}{y}$
6. $\frac{5a+b}{5a+c}$ (can't reduce)
7. -1
8. $\frac{4(x+2)}{x-2}$
9. 32
10. $3xy$
11. $(x-1)(x+3)$
12. $(1+b)(2-a)$
13. 2
14. $\frac{6}{9}$, $\frac{2}{9}$
15. $\frac{3}{x}$, $\frac{5x}{x}$
16. $\frac{x(x+1)}{3(x+1)}$, $\frac{-12}{3(x+1)}$
17. $\frac{3}{x-2}$, $\frac{-4}{x-2}$
18. $\frac{2x(x-1)}{30(x^2-2)(x-1)}$, $\frac{21x(x^2-2)(y-1)}{30(x^2-2)(x-1)}$
19. $\frac{6}{7}$
20. -1
21. $\frac{2b-2a}{b+a}$
22. 0
23. $\frac{5}{2a}$
24. $\frac{3x^2+2x}{x^2-4}$
25. $\frac{-3(2x-1)}{(x+1)(x-2)}$
26. 0
27. $\frac{b}{42}$
28. $\frac{3y^2}{x-4}$
29. $\frac{(a+b)(5-p)}{x-y}$
30. $\frac{9}{ab}$
31. $\frac{x+7}{x+3}$
32. $\frac{a^2-4a}{3-2a}$
33. $4a-2b$

TOPIC 2: OPERATIONS with EXPONENTS and RADICALS

A. Definitions of powers and roots:

Problems 1-20: Find the value:

1. $2^3 =$
2. $3^2 =$
3. $-4^2 =$
4. $(-4)^2 =$
5. $0^4 =$
6. $1^4 =$
7. $\sqrt{64} =$
8. $\sqrt[3]{64} =$
9. $\sqrt[6]{64} =$
10. $-\sqrt{49} =$
11. $\sqrt[3]{-125} =$
12. $\sqrt{5^2} =$
13. $\sqrt{(-5)^2} =$
14. $\sqrt{x^2} =$
15. $\sqrt[3]{a^3} =$
16. $\sqrt{\frac{1}{4}} =$
17. $\sqrt{.04} =$
18. $\left(\frac{2}{3}\right)^4 =$
19. $\sqrt[4]{81a^8} =$
20. $\sqrt{7} \bullet \sqrt{7} =$

B. Laws of integer exponents:

I. $a^b \bullet a^c = a^{b+c}$

II. $\frac{a^b}{a^c} = a^{b-c}$

III. $\left(a^b\right)^c = a^{bc}$

IV. $(ab)^c = a^c \bullet b^c$

V. $\left(\frac{a}{b}\right)^c = \frac{a^c}{b^c}$

VI. $a^0 = 1$ (if $a \neq 0$)

VII. $a^{-b} = \frac{1}{a^b}$

Problems 21-30: Find x:

21. $2^3 \bullet 2^4 = 2^x$
22. $\frac{2^3}{2^4} = 2^x$
23. $3^{-4} = \frac{1}{3^x}$
24. $\frac{5^2}{5^2} = 5^x$
25. $\left(2^3\right)^4 = 2^x$
26. $8 = 2^x$

27. $a^3 \bullet a = a^x$

28. $\frac{b^{10}}{b^5} = b^x$

29. $\frac{1}{c^{-4}} = c^x$

30. $\frac{a^{3y-2}}{a^{2y-3}} = a^x$

Problems 31-43: Find the value:

31. $7x^0 =$
32. $3^{-4} =$
33. $2^3 \bullet 2^4 =$
34. $0^5 =$
35. $5^0 =$
36. $(-3)^3 - 3^3 =$
37. $x^{c+3} \bullet x^{c-3} =$
38. $2^x \bullet 4^{x-1} =$
39. $\frac{x^{c+3}}{x^{c-3}} =$
40. $\frac{8x}{2^{x-1}} =$
41. $\frac{2x^{-3}}{6x^{-4}} =$
42. $\left(a^{x+3}\right)^{x-3} =$
43. $\frac{a^{3x-2}}{a^{2x-3}} =$

Problems 44-47: Write two given ways:

Given	***No negative powers***	***No fraction***
44. $\frac{d^{-4}}{d^4}$		
45. $\left(\frac{3x^3}{y}\right)^{-2}$		
46. $\left(\frac{a^2bc}{2ab^2c}\right)^3$		
47. $\frac{x^2y^3z^{-1}}{x^5y^{-6}z^{-3}}$		

C. <u>Laws of rational exponents, radicals</u>:
Assume all radicals are real numbers:

I. If *r* is a positive integer, *p* is an integer, and $a \ge 0$, then $a^{p/r} = \sqrt[r]{a^p} = \left(\sqrt[r]{a}\right)^p$, which is a real number. (Also true if *r* is a positive odd integer and $a < 0$).

II. $\sqrt[r]{ab} = \sqrt[r]{a} \bullet \sqrt[r]{b}$, or $(ab)^{1/r} = a^{1/r} \bullet b^{1/r}$

III. $\sqrt[r]{\frac{a}{b}} = \frac{\sqrt[r]{a}}{\sqrt[r]{b}}$, or $\left(a/b\right)^{1/r} = \frac{a^{1/r}}{b^{1/r}}$

IV. $\sqrt[rs]{a} = \sqrt[r]{\sqrt[s]{a}} = \sqrt[s]{\sqrt[r]{a}}$, or $a^{1/rs} = \left(a^{1/s}\right)^{1/r} = \left(a^{1/r}\right)^{1/s}$

Problems 48-53: Write as a radical:

48. $3^{1/2} =$
49. $4^{2/3} =$
50. $\left(\frac{1}{2}\right)^{1/3} =$
51. $x^{3/2} =$
52. $2x^{1/2} =$
53. $(2x)^{1/2} =$

Problems 54-57: Write as a fractional power:

54. $\sqrt{5} =$
55. $\sqrt{2^3} =$
56. $\sqrt[3]{a} =$
57. $\frac{1}{\sqrt{a}} =$

Problems 58-62: Find *x*:

58. $\sqrt{4} \bullet \sqrt{9} = \sqrt{x}$
59. $\sqrt{x} = \frac{\sqrt{4}}{\sqrt{9}}$
60. $\sqrt{\sqrt[3]{64}} = \sqrt{x}$
61. $\sqrt[3]{\sqrt{64}} = x$
62. $x = \frac{8^{2/3}}{4^{3/2}}$

Problems 63-64: Write with positive exponents:

63. $\left(9x^6y^{-2}\right)^{1/2} =$
64. $\left(-8a^6b^{-12}\right)^{-2/3} =$

D. <u>Simplification of radicals</u>:

example: $\sqrt{32} = \sqrt{16} \bullet \sqrt{2} = 4\sqrt{2}$
example: $\sqrt[3]{72} = \sqrt[3]{8} \bullet \sqrt[3]{9} = 2\sqrt[3]{9}$
example: $\sqrt[3]{54} + 4\sqrt[3]{16} = \sqrt[3]{27} \bullet \sqrt[3]{2} + 4\sqrt[3]{8} \bullet \sqrt[3]{2}$
$= 3\sqrt[3]{2} + 4 \bullet 2\sqrt[3]{2}$
$= 3\sqrt[3]{2} + 8\sqrt[3]{2} = 11\sqrt[3]{2}$
example: $\sqrt{8} - \sqrt{2} = 2\sqrt{2} - \sqrt{2} = \sqrt{2}$

Problems 65-78: Simplify (assume all radicals are real numbers):

65. $-\sqrt{81} =$
66. $\sqrt{50} =$
67. $3\sqrt{12} =$
68. $\sqrt[3]{54} =$
69. $\sqrt{52} =$
70. $2\sqrt{3} + \sqrt{27} - \sqrt{75} =$
71. $\sqrt{x^5} =$
72. $\sqrt{4x^6} =$
73. $x\sqrt{2x} + 2\sqrt{2x^3} + \frac{2x^2}{\sqrt{2x}} =$
74. $\sqrt{a^2} =$
75. $\sqrt{a^3} =$
76. $\sqrt[3]{a^5} =$
77. $3\sqrt{2} + \sqrt{2} =$
78. $5\sqrt{3} - \sqrt{3} =$

E. <u>Rationalization of denominators</u>:

example: $\sqrt{\frac{5}{8}} = \frac{\sqrt{5}}{\sqrt{8}} = \frac{\sqrt{5}}{\sqrt{8}} \bullet \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{10}}{\sqrt{16}} = \frac{\sqrt{10}}{4}$
example: $\frac{1}{\sqrt[3]{2}} = \frac{1}{\sqrt[3]{2}} \bullet \frac{\sqrt[3]{4}}{\sqrt[3]{4}} = \frac{\sqrt[3]{4}}{\sqrt[3]{8}} = \frac{\sqrt[3]{4}}{2}$
example: $\frac{\sqrt{3}}{\sqrt{3}-1} = \frac{\sqrt{3}}{\sqrt{3}-1} \bullet \frac{\sqrt{3}+1}{\sqrt{3}+1} = \frac{\sqrt{9}+\sqrt{3}}{3-1} = \frac{3+\sqrt{3}}{2}$

Problems 79-87: Simplify:

79. $\sqrt{\frac{2}{3}} =$
80. $\frac{1}{\sqrt{5}} =$
81. $\frac{3}{\sqrt{3}} =$
82. $\frac{\sqrt{a}}{\sqrt{b}} =$
83. $\sqrt[3]{\frac{2}{3}} =$
84. $\sqrt{2} + \frac{1}{\sqrt{2}} =$
85. $\frac{3}{\sqrt{2}+1} =$
86. $\frac{\sqrt{3}}{1-\sqrt{3}} =$
87. $\frac{\sqrt{3}+2}{\sqrt{3}-2} =$

<u>Answers</u>:

1. 8
2. 9
3. -16
4. 16
5. 0
6. 1
7. 8
8. 4
9. 2

10. -7
11. -5
12. 5
13. 5
14. x if $x \ge 0$; $-x$ if $x < 0$
15. a
16. $1/2$
17. .2
18. $16/81$
19. $3a^2$
20. 7
21. 7
22. -1
23. 4
24. 0
25. 12
26. 3
27. 4
28. 5
29. 4
30. $y+1$
31. 7
32. $1/81$
33. 128
34. 0
35. 1
36. -54
37. x^{2c}
38. 2^{3x-2}
39. x^6
40. 2^{2x+1}
41. $x/3$
42. a^{x^2-9}
43. a^{x+1}
44. $\frac{1}{d^8} = d^{-8}$
45. $\frac{y^2}{9x^6} = 9^{-1}x^{-6}y^2$
46. $\frac{a^3}{8b^3} = 8^{-1}a^3b^{-3}$
47. $\frac{y^9z^2}{x^3} = x^{-3}y^9z^2$
48. $\sqrt{3}$
49. $\sqrt[3]{16}$
50. $\sqrt[3]{1/2} = \sqrt[3]{4}/2$
51. $\sqrt{x^3} = x\sqrt{x}$
52. $2\sqrt{x}$
53. $\sqrt{2x}$
54. $5^{1/2}$
55. $2^{3/2}$
56. $a^{1/3}$
57. $a^{-1/2}$
58. 36
59. $4/9$
60. 4
61. 2
62. $1/2$
63. $\frac{3|x|^3}{|y|}$
64. $\frac{b^8}{4a^4}$
65. -9
66. $5\sqrt{2}$
67. $6\sqrt{3}$
68. $3\sqrt[3]{2}$
69. $2\sqrt{13}$
70. 0
71. $x^2\sqrt{x}$
72. $2|x|^3$
73. $4x\sqrt{2x}$
74. a if $a \ge 0$; $-a$ if $a < 0$
75. $a\sqrt{a}$
76. $a\sqrt[3]{a^2}$
77. $4\sqrt{2}$
78. $4\sqrt{3}$
79. $\sqrt{6}/3$
80. $\sqrt{5}/5$
81. $\sqrt{3}$
82. $\sqrt{ab}/b$
83. $\sqrt[3]{18}/3$
84. $3\sqrt{2}/2$
85. $3\sqrt{2}-3$
86. $\frac{\sqrt{3}+3}{-2}$
87. $-7-4\sqrt{3}$

TOPIC 3: LINEAR EQUATIONS and INEQUALITIES

A. Solving linear equations:
Add or subtract the same value on each side of the equation, or multiply or divide each side by the same value, with the goal of placing the variable alone on one side. If there are one or more fractions, it may be desirable to eliminate them by multiplying both sides by the common denominator. If the equation is a proportion, you may wish to cross-multiply.

Problems 1-15: Solve:

1. $2x = 9$
2. $3 = \frac{6x}{5}$
3. $3x + 7 = 6$
4. $\frac{x}{3} = \frac{5}{4}$
5. $5 - x = 9$
6. $x = \frac{2x}{5} + 1$
7. $4x - 6 = x$
8. $\frac{x-1}{x+1} = \frac{6}{7}$
9. $x - 4 = \frac{x}{2} + 1$
10. $\frac{3x}{2x+1} = \frac{5}{2}$
11. $6 - 4x = x$
12. $\frac{3x-2}{2x+1} = 4$
13. $\frac{x+3}{2x-1} = 2$
14. $7x - 5 = 2x + 10$
15. $\frac{1}{3} = \frac{x}{x+8}$

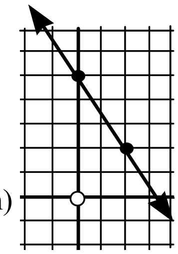
B. Solving a pair of linear equations:
The solution consists of an ordered pair, an infinite number of ordered pairs, or no solution.

Problems 16-23: Solve for the common solution(s) by substitution or linear combinations:

16. $x + 2y = 7$
    $3x - y = 28$
17. $x + y = 5$
    $x - y = -3$
18. $2x - y = -9$
    $x = 8$
19. $2x - y = 1$
    $y = x - 5$
20. $2x - 3y = 5$
    $3x + 5y = 1$
21. $4x - 1 = y$
    $4x + y = 1$
22. $x + y = 3$
    $x + y = 1$
23. $2x - y = 3$
    $6x - 9 = 3y$

C. Analytic geometry of one linear equation:
The graph of $y = mx + b$ is a line with slope m and y-intercept b. To draw the graph, find one point on it (such as (0, b)) and then use the slope to find another point. Draw the line joining the two points.

example: $y = \frac{-3}{2}x + 5$
has slope $-3/2$ and y-intercept 5.
To graph the line, locate (0, 5).
From that point, go down 3 (top of slope fraction), and over (right) 2 (bottom of fraction) to find a second point.
Draw the line joining the points.



Problems 24-28: Find slope and y-intercept, and sketch graph:

24. $y = x + 4$
25. $y = -\frac{1}{2}x - 3$
26. $2y = 4x - 8$
27. $x - y = -1$
28. $x = -3y + 2$

To find an equation of a non-vertical line, it is necessary to know its slope and one of its points. Write the slope of the line through (x, y) and the known point, then write an equation which says that this slope equals the known slope.

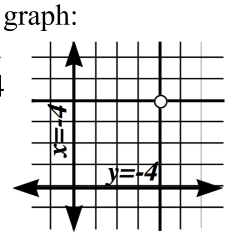
example: Find an equation of the line through (-4, 1) and (-2, 0).
Slope $= \frac{1-0}{-4+2} = \frac{1}{-2}$
Using (-2, 0) and (x, y),
Slope $= \frac{y-0}{x+2} = \frac{1}{-2}$; cross multiply,
get $-2y = x + 2$, or $y = -\frac{1}{2}x - 1$

Problems 29-33: Find an equation of line:

29. Through (-3, 1) and (-1, -4)
30. Through (0, -2) and (-3, -5)
31. Through (3, -1) and (5, -1)
32. Through (8, 0), with slope –1
33. Through (0, -5), with slope $2/3$

A vertical line has no slope, and its equation can be written so it looks like $x = k$ (where k is a number). A horizontal line has zero slope, and its equation looks like $y = k$.

example: Graph on the same graph:
$x - 3 = 1$ and $1 + y = -3$.
The first equation is $x = -4$
The second is $y = -4$



Problems 34-35: Graph and write equation for…

34. The line through (-1, 4) and (-1, 2)
35. The horizontal line through (4, -1)

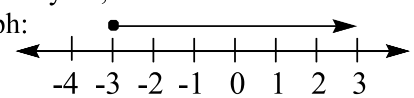
D. <u>Analytic geometry of two linear equations</u>:
Two distinct lines in a plane are either parallel or intersecting. They are parallel if and only if they have the same slope, and hence the equations of the lines have no common solutions. If the lines have unequal slopes, they intersect in one point and their equations have exactly one common solution. (They are perpendicular if their slopes are negative reciprocals, or one is horizontal and the other is vertical.) If one equation is a multiple of the other, each equation has the same graph, and every solution of one equation is a solution of the other.

Problems 36-43: For each pair of equations in problems 16 to 23, tell whether the lines are parallel, perpendicular, intersecting but not perpendicular, or the same line:

36. Problem 16
37. Problem 17
38. Problem 18
39. Problem 19
40. Problem 20
41. Problem 21
42. Problem 22
43. Problem 23

E. <u>Linear inequalities</u>:

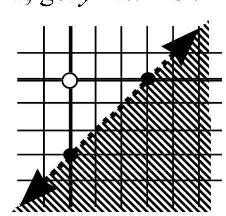
example: One variable graph: solve and graph on a number line: $1 - 2x \le 7$
(This is an abbreviation for $\{x\!: 1 - 2x \le 7\}$)
Subtract 1, get $-2x \le 6$
Divide by –2, $x \ge -3$
Graph:



Problems 44-50: Solve and graph on a number line:

44. $x - 3 > 4$
45. $4x < 2$
46. $2x + 1 \le 6$
47. $3 < x - 3$
48. $4 - 2x < 6$
49. $5 - x > x - 3$
50. $x > 1 + 4$

example: Two variable graph: graph solution on a number plane: $x - y > 3$
(This is an abbreviation for $\{(x, y) : x - y > 3\}$)
Subtract x, multiply by –1, get $y < x - 3$.
Graph $y = x - 3$,
but draw a dotted line, and shade the side where $y < x - 3$:



Problems 51-56: Graph on a number plane:

51. $y < 3$
52. $y > x$
53. $y \ge \frac{2}{3}x + 2$
54. $x < y + 1$
55. $x + y < 3$
56. $2x - y > 1$

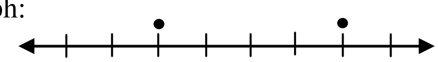
F. <u>Absolute value equations and inequalities</u>:

example: $|3-x|=2$

Since the absolute value of both 2 and –2 is 2, $3-x$ can be either 2 or –2. Write these two equations and solve each:

$$\begin{aligned} 3-x&=2 \\ -x&=-1 \\ x&=1 \end{aligned} \qquad \text{or} \qquad \begin{aligned} 3-x&=-2 \\ -x&=-5 \\ x&=5 \end{aligned}$$

Graph:



Problems 57-61: Solve and graph on number line:

57. $|x|=3$
58. $|x|=-1$
59. $|x-1|=3$
60. $|2-3x|=0$
61. $|x+2|=1$

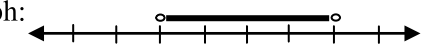
example: $|3-x|<2$

The absolute value of an number between –2 and 2 (exclusive) is less than 2. Write this inequality and solve: $-2<3-x<2$.

Subtract 3

Multiply by –1, get $5>x>1$. (Note that this says $x>1$ <u>and</u> $x<5$).

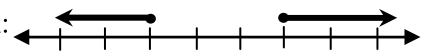
Graph:



example: $|2x+1|\ge 3$. The absolute value is greater than or equal to 3 for any number ≥ 3 <u>or</u> ≤ -3. So,

$$\begin{aligned} 2x+1&\ge 3 \\ 2x&\ge 2 \\ x&\ge 1 \end{aligned} \qquad \text{or} \qquad \begin{aligned} 2x+1&\le -3 \\ 2x&\le -4 \\ x&\le -2 \end{aligned}$$

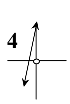
Graph:

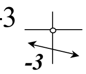


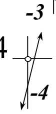
Problems 62-66: Solve and graph on number line:

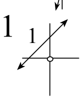
62. $|x|<3$
63. $3<|x|$
64. $|x+3|<1$
65. $1\le|x+3|$
66. $|5-x|<5$

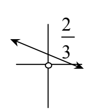
<u>Answers</u>:

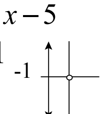
1. $\frac{9}{2}$
2. $\frac{5}{2}$
3. $-\frac{1}{3}$
4. $\frac{15}{4}$
5. –4
6. $\frac{5}{3}$
7. 2
8. 13
9. 10
10. $-\frac{5}{4}$
11. $\frac{6}{5}$
12. $-\frac{6}{5}$
13. $\frac{5}{3}$
14. 3
15. 4
16. (9, -1)
17. (1, 4)
18. (8, 25)
19. (-4, -9)
20. $\left(\frac{28}{19}, -\frac{13}{19}\right)$
21. $\left(\frac{1}{4}, 0\right)$
22. no solution
23. $(a,\ 2a-3)$, where a is any number; infinite number of solutions
24. $m=1,\ b=4$ 

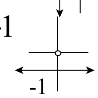
25. $m=-\frac{1}{2},\ b=-3$ 

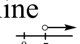
26. $m=2,\ b=-4$ 

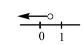
27. $m=1,\ b=1$ 

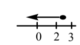
28. $m=-\frac{1}{3},\ b=\frac{2}{3}$ 

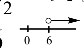
29. $y=-\frac{5}{2}x-\frac{13}{2}$
30. $y=x-2$
31. $y=-1$
32. $y=-x+8$
33. $y=\frac{2}{3}x-5$
34. $x=-1$ 

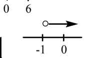
35. $y=-1$ 

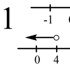
36. intersecting, not$\perp$
37. $\perp$
38. intersecting, not$\perp$
39. intersecting, not$\perp$
40. intersecting, not$\perp$
41. intersecting, not$\perp$
42. parallel
43. same line
44. $x>7$ 

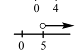
45. $x<\frac{1}{2}$ 

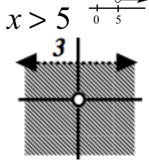
46. $x\le\frac{5}{2}$ 

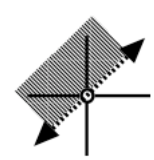
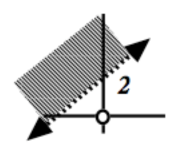
47. $x>6$ 

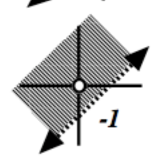
48. $x>-1$ 

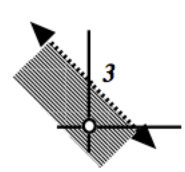
49. $x<4$ 

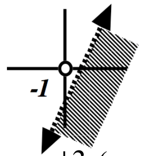
50. $x>5$ 

51. 

52. 
53. 

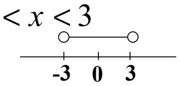
54. 

55. 

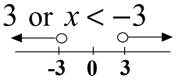
56. 

57. $x = \pm 3$ (on number line)
58. no solution
59. $x = -2,\ 4$

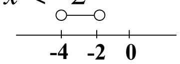
60. $x = \frac{2}{3}$
61. $x = -1,\ -3$
62. $-3 < x < 3$



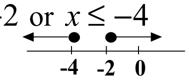
63. $x > 3$ or $x < -3$



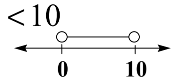
64. $-4 < x < -2$



65. $x \geq -2$ or $x \leq -4$



66. $0 < x < 10$



TOPIC 4: POLYNOMIALS and POLYNOMIAL EQUATIONS

A. <u>Solving quadratic equations by factoring</u>:
If $ab = 0$, then $a = 0$ or $b = 0$

> *example:* If $(3-x)(x+2)=0$
> then $(3-x)=0$ or $(x+2)=0$ and thus
> $x = 3$ or $x = -2$

Note: there must be a zero on one side of the equation to solve by the factoring method.

> *example:* $6x^2 = 3x$
> Rewrite: $6x^2 - 3x = 0$
> Factor: $3x(2x-1)=0$
> So $3x = 0$ or $(2x-1)=0$.
> Thus, $x = 0$ or $x = \frac{1}{2}$.

Problems 1-12: Solve by factoring:

1. $x(x-3)=0$	7. $x^2 - x - 6 = 0$
2. $x^2 - 2x = 0$	8. $x^2 = 2 - x$
3. $2x^2 = x$	9. $6x^2 = x + 2$
4. $3x(x+4)=0$	10. $x^2 + x = 6$
5. $(x+2)(x-3)=0$	11. $9 + x^2 = 6x$
6. $(2x+1)(3x-2)=0$	12. $1 - x = 2x^2$

B. <u>Monomial factors</u>:
The distributive property says $ab + ac = a(b+c)$

> *example:* $x^2 - x = x(x-1)$
> *example:* $4x^2y + 6xy = 2xy(2x+3)$

Problems 13-17: Factor:

13. $a^2 + ab$	16. $x^2y - y^2x =$
14. $a^3 - a^2b + ab^2 =$	17. $6x^3y^2 - 9x^4y =$
15. $-4xy + 10x^2$	

C. <u>Factoring: $(x-a)(x+b)^2 + (x-a)(x+c)$</u>:
The distributive property says $jm + jn = j(m+n)$.
Compare this equation with the following:

$$(x+1)(x+3)^2 + (x+1)(x-4)$$
$$= (x+1)\left((x+3)^2 + (x-4)\right)$$

Note that $j = x+1$, $m = (x+3)^2$, and $n = (x-4)$, and we get $(x+1)\left(x^2 + 6x + 9 + x - 4\right)$

$$= (x+1)\left(x^2 + 7x + 5\right)$$

Problems 18-20: Find *P* which completes the equation:

18. $(x-2)(x-1)^2 - (x-2)(x+3)$
 $= (x-2)(\quad P \quad)$
19. $(x+4)(x-3)^2 + (x-4)(x-3)$
 $= (x-3)(\quad P \quad)$
20. $(2x-1)(x+1) - (2x-1)^2(x+3)$
 $= (2x-1)(\quad P \quad)$

D. <u>The quadratic formula</u>:
If a quadratic equation looks like $ax^2 + bx + c = 0$, then the roots (solutions) can be found by using the quadratic formula: $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

> *example:* $3x^2 + 2x - 1 = 0$, $a = 3$, $b = 2$, and $c = -1$
> $x = \frac{-2 \pm \sqrt{2^2 - 4(3)(-1)}}{2 \bullet 3} = \frac{-2 \pm \sqrt{4+12}}{6} = \frac{-2 \pm \sqrt{16}}{6}$
> $= \frac{-2 \pm 4}{6} = \frac{-6}{6} = -1$ or $\frac{2}{6} = \frac{1}{3}$
> *example:* $x^2 - x - 1 = 0$, $a = 1$, $b = -1$, $c = -1$
> $x = \frac{1 \pm \sqrt{1+4}}{2} = \frac{1 \pm \sqrt{5}}{2}$ So there are two roots:
> $\frac{1+\sqrt{5}}{2}$ and $\frac{1-\sqrt{5}}{2}$

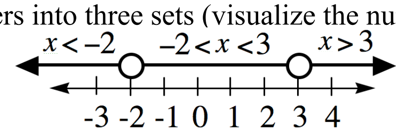
Problems 21-24: Solve:

21. $x^2 - x - 6 = 0$	23. $2x^2 - x - 2 = 0$
22. $x^2 + 2x = -1$	24. $x^2 - 3x - 4 = 0$

E. <u>Quadratic inequalities</u>:

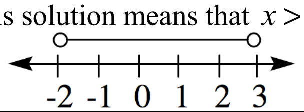
> *example:* Solve $x^2 - x < 6$. First make one side zero: $x^2 - x - 6 < 0$.
> Factor: $(x-3)(x+2) < 0$.
> If $(x-3)=0$ or $(x+2)=0$, then $x = 3$ or $x = -2$.

These two numbers (3 and –2) split the real numbers into three sets (visualize the number line):



x	$(x-3)$	$(x+2)$	$(x-3)(x+2)$	solution?
$x<-2$	negative	negative	positive	no
$-2<x<3$	negative	positive	negative	yes
$x>3$	positive	positive	positive	no

Therefore, if $(x-3)(x+2)<0$, then $-2<x<3$
Note that this solution means that $x>-2$ <u>and</u> $x<3$



Problems 25-29: Solve, and graph on a number line:

25. $x^2-x-6>0$
26. $x^2+2x<0$
27. $x^2-2x<-1$
28. $x>x^2$
29. $2x^2+x-1>0$

F. <u>Completing the square</u>:

x^2+bx will be the square of a binomial when c is added, if c is found as follows: find half the coefficient of x, and square it–this is c. Thus $c=\left(\frac{b}{2}\right)^2=\frac{b^2}{4}$, and x^2+bx+c
$=x^2+bx+\frac{b^2}{4}=\left(x+\frac{b}{2}\right)^2$

example: x^2+5x
Half of 5 is $5/2$, and $(5/2)^2=25/4$, which must be added to complete the square:
$x^2+5x+\frac{25}{4}=\left(x+\frac{5}{2}\right)^2$

If the coefficient of x^2 is not 1, factor so it is.

example: $3x^2-x=3\left(x^2-\frac{1}{3}x\right)$ Half of $-1/3$ is $-1/6$, and $(-1/6)^2=1/36$, so
$\left(x^2-\frac{1}{3}x+\frac{1}{36}\right)=\left(x-\frac{1}{6}\right)^2$, and
$3\left(x^2-\frac{1}{3}x+\frac{1}{36}\right)=3x^2-x+\frac{3}{36}$. Thus, $\frac{3}{36}$ (or $\frac{1}{12}$) must be added to $3x^2-x$ to complete the square.

Problems 30-33: Complete the square, and tell what must be added:

30. x^2-10x
31. x^2+x
32. $x^2-\frac{3}{2}x$
33. $2x^2+8x$

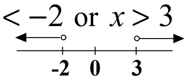
G. <u>Graphing quadratic functions</u>:

Problems 34-40: Sketch the graph:

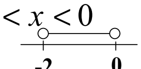
34. $y=x^2$
35. $y=-x^2$
36. $y=x^2+1$
37. $y=x^2-3$
38. $y=(x+1)^2$
39. $y=(x-2)^2-1$
40. $y=(x+2)(x-1)$

<u>Answers</u>:

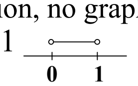
1. 0, 3
2. 0, 2
3. 0, $1/2$
4. 0, –4
5. –2, 3
6. $-1/2$, $2/3$
7. 3, –2
8. –2, 1
9. $2/3$, $-1/2$
10. –3, 2
11. 3
12. –1, $1/2$
13. $a(a+b)$
14. $a\left(a^2-ab+b^2\right)$
15. $2x(-2y+5x)$
16. $xy(x-y)$
17. $3x^3y(2y-3x)$
18. x^2-3x-2
19. $x^2+2x-16$
20. $-2x^2-4x+4$
21. –2, 3
22. –1
23. $\frac{1\pm\sqrt{17}}{4}$
24. –1, 4
25. $x<-2$ or $x>3$



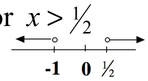
26. $-2<x<0$



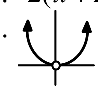
27. no solution, no graph
28. $0<x<1$



29. $x<-1$ or $x>1/2$



30. $(x-5)^2$, add 25
31. $\left(x+1/2\right)^2$, add $1/4$
32. $\left(x-3/4\right)^2$, add $9/16$
33. $2(x+2)^2$, add 8
34.



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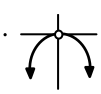

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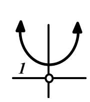

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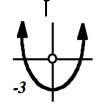

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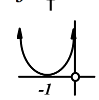
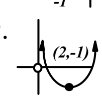


39.



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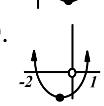


TOPIC 5: FUNCTIONS

A. What functions are and how to write them:
The area of a square depends on the side length *s*, and given *s*, we can find the area *A* for that value of *s*. The side and area can be thought of as an ordered pair: (s, A). For example, (5, 25) is an ordered pair. Think of a function as a set of ordered pairs with one restriction: no two different ordered pairs may have the same first element. Thus $\{(s, A) : A$ is the area of the square with side length $s\}$ is a function consisting of an infinite set of ordered pairs.

A function can also be thought of as a rule: for example, $A = s^2$ is the rule for finding the area of a square, given a side. The area depends on the given side and we say the area is a function of the side. '$A = f(s)$' is read 'A is a function of s', or '$A = f$ of s'. There are many functions of s , the one here is s^2. We write this $f(s) = s^2$ and can translate: 'the function of *s* is s^2'. Sometimes we write $A(s) = s^2$. This says the area is a function of *s*, and specifically, it is s^2.

In some relations, as $x^2 + y^2 = 25$, *y* is not a function of *x*, since both (3, 4) and (3, -4) make the relation true.

Problems 1-7: Tell whether or not each set of ordered pairs is a function:

1. $\{(1,3)(-1,3)(0,-1)\}$
2. $\{(3,1)(3,-1)(-1,0)\}$
3. $\{(0,5)\}$
4. $\{(x, y) : y = x^2$ and x is any real number$\}$
5. $\{(x, y) : x = y^2$ and y is any real number$\}$
6. $\{(x, y) : y = -3$ and x is any real number$\}$
7. $\{(x, y) : x = 4$ and y is any real number$\}$

Problems 8-11: Is *y* a function of *x*?

8. $y = 4x + x^2$
9. $y^2 = x^2$
10. $(x + y)^2 = 36$
11. $y = \sqrt{x}$

B. Function values and substitution:
If $A(s) = s^2$, $A(3)$, read 'A of 3', means replace every s in $A(s) = s^2$ with 3, and find the area when s is 3. When we do this, we find $A(3) = 3^2 = 9$.

example: $g(x)$ is given: $y = g(x) = \pi x^2$
example: $g(3) = \pi \bullet 3^2 = 9\pi$
example: $g(7) = \pi \bullet 7^2 = 49\pi$
example: $g(a) = \pi a^2$
example: $g(x + h) = \pi(x + h)^2 = \pi x^2 + 2\pi xh + \pi h^2$

12. Given $y = f(x) = 3x - 2$; complete these ordered pairs:
(3, __), (0, __), ($\frac{1}{2}$, __), (__, 10), (__, -1), ($x - 1$, ___)

Problems 13-17: Given $f(x) = \frac{x}{x+1}$ Find:

13. $f(1) =$
14. $f(-2) =$
15. $f(0) =$
16. $f(-1) =$
17. $f(x - 1) =$

C. Composition of functions:

example: If $f(x) = x^2$, and $g(x) = x - 3$, $f(g(x))$ is read '*f* of *g* of *x*', and means replace every *x* in $f(x) = x^2$ with $g(x)$ giving $f(g(x)) = (g(x))^2$, which equals $(x - 3)^2 = x^2 - 6x + 9$.
example: $g(f(\frac{1}{2})) = g((\frac{1}{2})^2)$
$= g(\frac{1}{4}) = \frac{1}{4} - 3 = -2\frac{3}{4}$

Problems 18-26: Use *f* and *g* as above:

18. $g(f(x)) =$
19. $f(g(1)) =$
20. $g(g(x)) =$
21. $f(x) + g(x) =$
22. $f(x) \bullet g(x) =$
23. $f(x) - g(x) =$
24. $\frac{f(x)}{g(x)} =$
25. $g(x^2) =$
26. $(g(x))^2 =$

example: If $k(x) = x^2 - 4x$, for what *x* is $k(x) = 0$? If $k(x) = 0$, then $x^2 - 4x = 0$ and since $x^2 - 4x = x(x - 4) = 0$, *x* can be either 0 or 4. (These values of *x*: 0 and 4, are called 'zeros of the function', because each makes the function zero.)

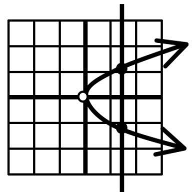
Problems 27-30: Find *x* so:

27. $k(x) = -4$
28. $k(x) = 5$
29. x is a zero of $x(x + 1)$
30. x is a zero of $(2x^2 - x - 3)$

D. Graphing functions:
An easy way to tell whether a relation between two variables is a function or not is by graphing it: if a vertical line can be drawn which has two or more

points in common with the graph, the relation is <u>not</u> a function. If <u>no</u> vertical line touches the graph more than once, then it <u>is</u> a function.

example: $x = y^2$ has this graph:



<u>Not</u> a function (the vertical line hits it more than once).

Problems 31-39: Tell whether or not each of the following is a function:

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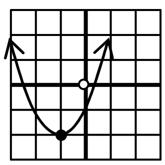

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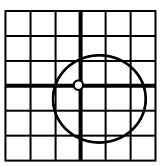

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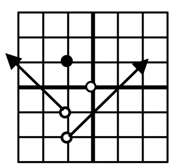

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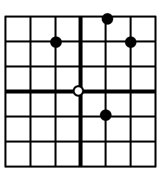

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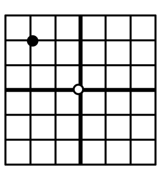

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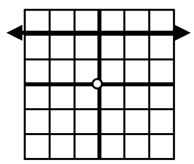

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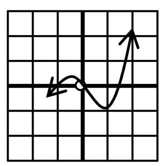

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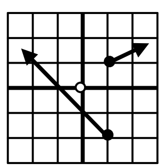

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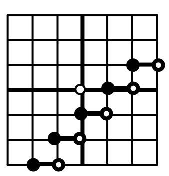
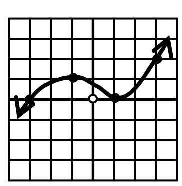
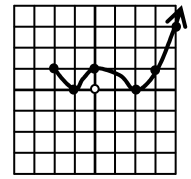


Since $y = f(x)$, the values of y are the values of the function, which correspond to specific values of x. The heights of the graph above (or below) the x-axis are the values of y and so also of the function. Thus for this graph, $f(3)$ is the height (value) of the function at $x = 3$ and value is 2: At $x = -3$, the value (height) of $f(x)$ is zero; in other words, $f(-3) = 0$. Note that $f(3) > f(-3)$, since $2 > 0$, and that $f(0) < f(-1)$, since $f(-1) = 1$ and $f(0) < 1$.



Problems 40-44: For this graph, tell whether the statement is true or false:



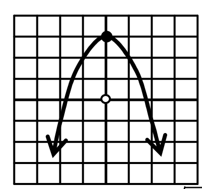
40. $g(-1) = g(0)$
41. $g(0) = g(3)$
42. $g(1) > g(-1)$
43. $g(-2) > g(1)$
44. $g(2) < g(0) < g(4)$

To graph $y = f(x)$, determine the degree of $f(x)$ if it is a polynomial. If it is linear (first degree) the graph is a line, and you merely plot two points (select any x and find the corresponding y) and draw their line.
If $f(x)$ is quadratic (second degree), its graph is a parabola, opening upward if the coefficient of x^2 is positive, downward if negative.
To plot <u>any</u> graph, it can be helpful to find the following:

a) The y-intercept (find $f(0)$ to locate y-axis crossing)
b) The x-intercept (find x for which $f(x) = 0$ – x-axis crossing)
c) What happens to y when x is very large (positive) or very small negative?
d) What happens to y when x is very close to a number which makes the bottom of a fraction zero?
e) Find x in terms of y, and find what happens to x as y approaches a number which makes the bottom of a fraction zero.

(*d*, *e*, and sometimes *c* above will help find vertical and horizontal asymptotes.)

example: $y = 3 - x^2$
analysis: a quadratic function (due to x^2)



y-intercept: $f(0) = 3$
x-intercepts: $y = 3 - x^2 = 0$ so $x = \pm\sqrt{3}$

example: $y = \frac{-2}{x+1}$

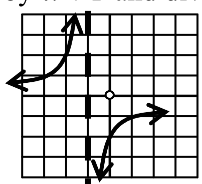
a) y-intercepts: $f(0) = -2$
b) x-intercept: none, since there is no solution to $y = \frac{-2}{x+1} = 0$
c) Large x: negative y approaches zero; very negative x makes y positive and going to zero. (So $y = 0$, the x-axis, is an asymptote line.)
d) The bottom of the fraction, $x + 1$, is zero if $x = -1$. As x moves to –1 from the left, y gets very large positive, and if x approaches –1 from above, y becomes very negative. (The line $x = -1$ is an asymptote.)

Graph:

e) To solve for x, multiply by $x+1$ and divide by y to get $x+1=-\frac{2}{y}$

$x=\frac{-2}{y}-1$

or $=\frac{-2-y}{y}$

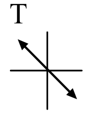
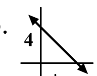


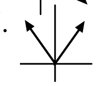
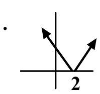
Note that y close to zero results in a very positive or negative x and means $y=0$ is an asymptote, which we already found above in part c.

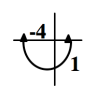
Problems 45-54: Sketch the graph:

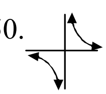
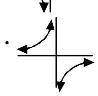
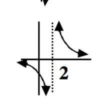
45. $f(x)=-x$
46. $y=4-x$
47. $y=|x|$
48. $y=|x-2|$
49. $y=x^2+3x-4$
50. $y=\frac{2}{x}$
51. $f(x)=\frac{-2}{x}$
52. $y=\frac{1}{x-2}$
53. $y=\frac{1}{x^2}$
54. $y=\frac{3}{x-1}$

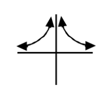
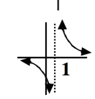
Answers:

1. yes
2. no
3. yes
4. yes
5. no
6. yes
7. no
8. yes
9. no (2 intersecting lines)
10. no (2 parallel lines)
11. yes
12. 7, -2, $-\frac{1}{2}$, 4, $\frac{1}{3}$, $3x-5$
13. $\frac{1}{2}$
14. 2
15. 0
16. none (undefined)
17. $\frac{x-1}{x}$
18. x^2-3
19. 4
20. $x-6$
21. x^2+x-3
22. x^3-3x^2
23. x^2-x+3
24. $\frac{x^2}{x-3}$
25. x^2-3
26. x^2-6x+9
27. 2
28. –1, 5
29. –1, 0
30. –1, $\frac{3}{2}$
31. yes
32. no
33. yes
34. no
35. yes
36. yes
37. yes
38. no
39. yes
40. F
41. T
42. T
43. T
44. T
45. 
46. 

47. 
48. 

49. 

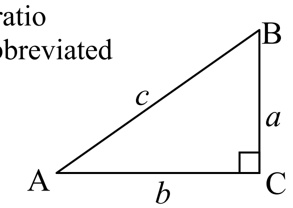
50. 
51. 
52. 

53. 
54. 

TOPIC 6: TRIGONOMETRY

A. <u>Trig functions in right triangles</u>:

The sine ratio for an acute angle of a right triangle is defined to be the length of the opposite leg to the length of the hypotenuse. Thus the sine ratio for angle B, abbreviated sin B, is $\frac{b}{c}$.



The reciprocal of the sine ratio is the cosecant (csc), so csc B$=\frac{c}{b}$.

The other four trig ratios (all functions) are

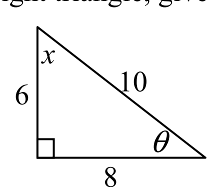
$$\text{cosine}=\cos=\frac{\textit{adjacent leg}}{\textit{hypotenuse}}$$

$$\text{secant}=\sec=\frac{1}{\cos}=\frac{\textit{hypotenuse}}{\textit{adjacent leg}}$$

$$\text{tangent}=\tan=\frac{\textit{opposite leg}}{\textit{adjacent leg}}$$

$$\text{cotangent}=\cot=\text{ctn}=\frac{\textit{adjacent leg}}{\textit{opposite leg}}$$

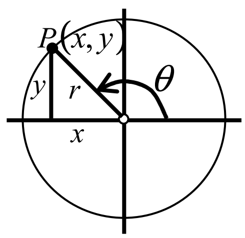
Problems 1-8: For this right triangle, give the following ratios:



1. $\tan x=$
2. $\frac{\sin x}{\cos x}=$
3. $\cos\theta=$

4. $\sin\theta \bullet \cos\theta =$
5. $\cos^2 x$, which means $(\cos x)^2 =$
6. $1 - \sin^2 x =$
7. $\frac{\cos x}{\sin\theta} =$
8. $\frac{\sin\theta}{\cos\theta} =$

B. <u>Circular trig definitions</u>:
Given a circle with radius *r*, centered on (0, 0). Draw the radius connecting a line from the vertex to any point on the circle, making an angle θ with the positive *x*-axis (θ may be any real number, positive measure is counter-clockwise). The coordinates (*x*, *y*) of point *P* together with radius *r* are used to define the functions:
$\sin\theta = y/r$, $\cos\theta = x/r$, $\tan\theta = y/x$, and the reciprocal functions as before. (Note that for $0 < \theta < \pi/2$, these definitions agree with the right triangle definitions. Also note that $-1 \le \sin\theta \le 1$, $-1 \le \cos\theta \le 1$, and $\tan\theta$ can be any real number.)



Problems 9-12: For the point (-3, 4) on the above circle, give:

9. $x =$ $\qquad y =$ $\qquad r =$
10. $\tan\theta =$
11. $\cos\theta =$
12. $\cot\theta \bullet \sin\theta =$

Note that for any given value of a trig function, (in its range), there are infinitely many values of θ.

Problems 13-14: Find two positive and two negative values θ for which:

13. $\sin\theta = -1$
14. $\tan\theta = \tan 45°$

Problems 15-16: Given $\sin\theta = 3/5$ and $\pi/2 < \theta < \pi$, then:

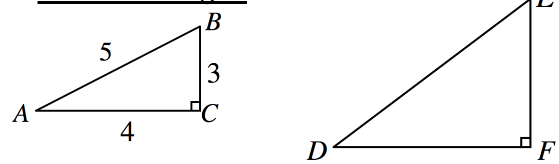
15. $\tan\theta =$
16. $\cos\theta =$

C. <u>Pythagorean relations (identities)</u>:
$a^2 + b^2 = c^2$ (or $x^2 + y^2 = r^2$)
above, can be divided by c^2 (or r^2)
to give $a^2/c^2 + b^2/c^2 = c^2/c^2$, or
$\sin^2 A + \cos^2 A = 1$, (or $\sin^2\theta + \cos^2\theta = 1$),
called an identity because it is true for <u>all</u> values of *A* for which it is defined.

Problems 17-18: Get a similar identity by dividing $a^2 + b^2 = c^2$ by:

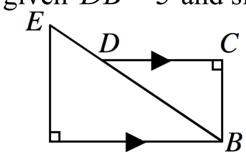
17. b^2
18. a^2

D. <u>Similar triangles</u>:

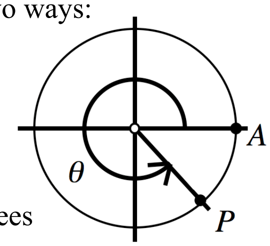


If $\Delta ABC \sim \Delta DEF$, and if $\tan A = 3/4$, then $\tan D = 3/4$ also, since $EF : DF = BC : AC = 3/4$.

19. Find *DC*, given $DB = 5$ and $\sin E = .4$



E. <u>Radians and degrees</u>:
For angle θ, there is a point *P* on the circle, and an arc from *A* counter-clockwise to *P*. The length of the arc is $\frac{\theta°}{360°} \bullet C = \frac{\theta}{360} \bullet 2\pi r$, and the ratio of the length of arc to radius is $\frac{\pi}{180} \bullet \theta$, where θ is the number of degrees (and the ratio has no units). This is the <u>radian</u> measure associated with point *P*. So *P* can be located two ways: by giving the central angle θ in degrees, <u>or</u> in a number of radii to be wrapped around the circle from point *A* (the radian measure).
Converting: radians $= \frac{\pi}{180} \bullet$ degrees
or degrees $= \frac{180}{\pi} \bullet$ radians.



> *example:* $\frac{\pi}{3}$ (radians) $= \frac{\pi}{3} \bullet \frac{180}{\pi} = \frac{180}{3} = 60°$
> *example:* $420° = 420 \bullet \frac{\pi}{180} = \frac{7}{3}\pi$(radians)
> (which means that it would take a little over 7 radii to wrap around the circle from *A* to 420°.)

Problems 20-23: Find the radian measure for a central angle of:

20. $36° =$
21. $-45° =$
22. $180° =$
23. $217° =$

Problems 24-26: Find the degree measure, which corresponds to radian measure of:

24. $\frac{3\pi}{2} =$
25. $-3 =$
26. $-\frac{7\pi}{6} =$

Problems 27-29: Find the following values by sketching the circle, central angle, and a vertical segment from point *P* to the *x*-axis. (Radian measure if no units are given.) Use no tables or calculator.

27. $\cos\frac{5\pi}{6} =$
28. $\tan(-315°) =$
29. $\sin(-225°) =$

Problems 30-33: Sketch to evaluate without table or calculator:

30. $\sec 180° =$
31. $\cot(-\frac{3\pi}{2}) =$
32. $\sin \pi =$
33. $\cos\frac{3\pi}{2} =$

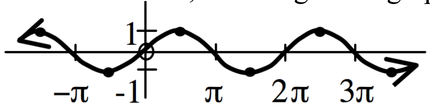
F. Trigonometric equations:

example: Solve, given that $0 \le \theta < 2\pi : \tan^2\theta - \tan\theta = 0$. Factoring, we get $\tan\theta(\tan\theta - 1) = 0$, which means that $\tan\theta = 0$ or $\tan\theta = 1$. Thus $\theta = 0$ (degrees or radians) plus any multiple of $180°$ (or π), which is $n \bullet 180°$ (or $n \bullet \pi$), or $\theta = 45°$ ($\pi/4$ radians), or $45 + n \bullet 180°$ (or $\pi/4 + n \bullet \pi$).
Thus, θ can be 0, $\pi/4$, π, or $5\pi/4$, which all check in the original equation.

Problems 34-39: Solve, for $0 \le \theta \le 2\pi$:

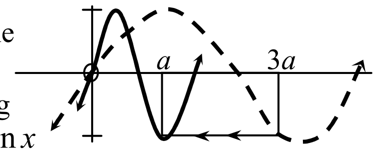
34. $\sin\theta = \cos\theta$
35. $\sin^2\theta = 1 + \cos\theta$
36. $\sin^2\theta = 1 - \cos^2\theta$
37. $\sin\theta \cot\theta = 1/2$
38. $\cos\theta \tan\theta = 1$
39. $\sin\theta \tan\theta = \sec\theta$

G. Graphs of trig functions:
By finding values of $\sin x$ when x is a multiple of $\pi/2$, we can get a quick sketch of $y = \sin x$. The sine is periodic (it repeats every 2π, its period). $|\sin x|$ never exceeds one, so the amplitude of $\sin x$ is 1, and we get this graph:



To graph $y = \sin 3x$, we note that for a given value of x, say $x = a$, the value of y is found on the graph of $y = \sin x$ three times as far from the y-axis as a.

Thus all points of the graph. $y = \sin 3x$ are found by moving each point of $y = \sin x$ graph to $1/3$ its previous distance from the y-axis, showing the new graph repeats 3 times in the period of $y = \sin x$, so the period of $y = \sin x$, so the period of $y = \sin 3x$ is $2\pi/3$.



Problems 40-46: Sketch each graph and find its period and amplitude:

40. $y = \cos x$
41. $y = \cos 2x$
42. $y = \tan x$
43. $y = \tan\frac{x}{3}$
44. $y = -4\sin\frac{2}{3}x$
45. $y = \sin^2 x$
46. $y = 1 + \cos x$

H. Identities:

example: Find a formula for cos2A, given
$\cos(A + B) = \cos A \cos B - \sin A \sin B$.
Substitute A for B:
$\cos 2A =$
$\cos(A + A) = \cos A \cos A - \sin A \sin A =$
$\cos^2 A - \sin^2 A$

Problems 47-49: Use $\sin^2 x + \cos^2 x = 1$ and the above to show:

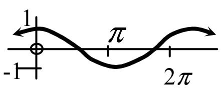
47. $\cos 2A = 2\cos^2 A - 1$
48. $\cos 2A = 1 - 2\sin^2 A$
49. $\cos^2 x = \frac{1}{2}(1 + \cos 2x)$

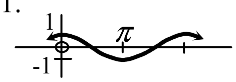
50. Given $\cos A = \frac{1}{\sec A}$, $\frac{\sin A}{\cos A} = \tan A$, and $\sin^2 A + \cos^2 A = 1$, show that $\tan^2 x + 1 = \sec^2 x$.

51. Given $\sin 2A = 2\sin A \cos A$, show $8\sin\frac{1}{2}x\cos\frac{1}{2}x = 4\sin x$.

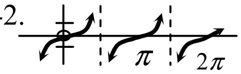
Answers:

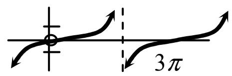
1. $4/3$
2. $4/3$
3. $4/5$
4. $12/25$
5. $9/25$
6. $9/25$
7. 1
8. $3/4$
9. $-3, 4, 5$
10. $-4/3$
11. $-3/5$
12. $-3/5$
13. $-5\pi/2, -\pi/2, 3\pi/2, 7\pi/2$
14. $-315°, -135°, 45°, 225°$
15. $-3/4$
16. $-4/5$
17. $\tan^2 A + 1 = \sec^2 A$
18. $1 + \cot^2 A = \csc^2 A$
19. 2
20. $\pi/5$
21. $-\pi/4$
22. π
23. $217\pi/180$
24. $270°$
25. $(-540/\pi)°$
26. $-210°$
27. $-\sqrt{3}/2$
28. 1
29. $\sqrt{2}/2$

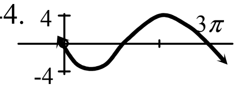
30. -1
31. 0
32. 0
33. 0
34. $\pi/4,\ 5\pi/4$
35. $\pi/2,\ 3\pi/2,\ \pi$
36. $0 \le \theta < 2\pi$
37. $\pi/3,\ 5\pi/3$
38. $\pi/2$
39. $\pi/2,\ 3\pi/2$
40. 

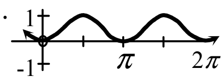
$P = 2\pi$, $A = 1$
41. 

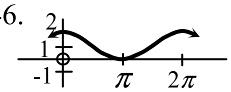
$P = \pi$, $A = 1$

42. 

$P = \pi$, *no* A
43. 

$P = 3\pi$, *no* A
44. 

$P = 3\pi$, $A = 4$
45. 

$P = \pi$, $A = 1/2$
46. 

$P = 2\pi$, $A = 1$

47. $\cos 2A = \cos^2 A - \sin^2 A$
$= \cos^2 A - (1 - \cos^2 A) = 2\cos^2 A - 1$
48. $\cos 2A = \cos^2 A - \sin^2 A$
$= (1 - \sin^2 A) - \sin^2 A = 1 - 2\sin^2 A$
49. From problem 47, $\cos 2x = 2\cos^2 x - 1$. Add 1, divide by 2.
50. $\tan^2 A + 1 = \frac{\sin^2 A}{\cos^2 A} + 1$
$= \frac{\sin^2 A + \cos^2 A}{\cos^2 A} = \frac{1}{\cos^2 A} = \sec^2 A$
51. Multiply given by 4, then let $2A = x$

TOPIC 7: LOGARITHMIC and EXPONENTIAL FUNCTIONS

A. <u>Logarithms and exponents</u>:

Exponential form: $2^3 = 8$
Logarithmic form: $\log_2 8 = 3$

Both of the equations above say the same thing. '$\log_2 8 = 3$' is read 'log base two of eight equals three' and translates 'the power of 2 which gives 8 is 3.'

Problems 1-4: Write the following information in both exponential and logarithmic forms:

1. The power of 3, which gives 9 is 2.
2. The power of x, which gives x^3 is 3.
3. 10 to the power -2 is $\frac{1}{100}$
4. $\frac{1}{2}$ is the power of 169 which gives 13.

<u>exponent rules</u>: (all quantities real)	<u>log rules</u>: (base any positive real number except 1)
$a^b \bullet a^c = a^{b+c}$	$\log ab = \log a + \log b$
$\frac{a^b}{a^c} = a^{b-c}$	$\log \frac{a}{b} = \log a - \log b$
$\left(a^b\right)^c = a^{bc}$	$\log a^b = b \log a$
$(ab)^c = a^c b^c$	$\log_a a^b = b$
$\left(\frac{a}{b}\right)^c = \frac{a^c}{b^c}$	$a^{(\log_a b)} = b$
$a^0 = 1\ (if\ a \ne 0)$	$\log_a b = \frac{\log_c b}{\log_c a}$
$a^{-b} = \frac{1}{a^b}$	
$a^{\frac{p}{r}} = \sqrt[r]{a^p} = \left(\sqrt[r]{a}\right)^p$	
(think of $\frac{p}{r}$ as $\frac{\text{power}}{\text{root}}$)	(base change rule)

Problems 5-25: Use the exponent and log rules to find the value of x:

5. $4^3 \bullet 4^5 = 4^x$
6. $3^{-2} = x$
7. $\frac{3^x}{3} = 3^0$
8. $\left(5^2\right)^3 = 5^x$
9. $\log_3 3^7 = x$
10. $6^{\log 6 x} = 8$
11. $\log_{10} x = \log_{10} 4 + \log_{10} 2$
12. $\log_x 25 = 2$
13. $\log_2 \frac{1}{32} = x$
14. $\log_{16} x = \frac{3}{2}$
15. $3\log_a 4 = \log_a x$
16. $\frac{\log_x 3}{\log_x 4} = \log_4 3$
17. $\log_3(27 \bullet 3^{-4}) = x$
18. $\log(2x - 6) = \log(6 - x)$
19. $\log_4 64 = x$
20. $\sqrt[4]{5} = 5^x$
21. $\frac{\log_3 81}{\log_3 27} - \log_3 \frac{81}{27} = x$
22. $\log_4 \sqrt[3]{30} = x \log_4 30$
23. $27^x = \left(\frac{1}{9}\right)^3$
24. $4^{10} = 2^x$
25. $3 \bullet 2^x = 4$

Problems 26-31: Find the value:

26. $2^{10} =$
27. $\log_4 4^{10} =$
28. $\log_6 6 =$
29. $\frac{6^{10}}{3^{10}} =$
30. $\log_{49} 7 =$
31. $\log_7 49 =$

32. Find $\log_3 2$, given: $\log_{10} 3 = .477$, $\log_{10} 2 = .301$

Problems 33-34: Given $\log_2 1024 = 10$, find:

33. $\log_2 1024^5 =$
34. $\log_2 \sqrt{1024} =$

Problems 35-46: Solve for x in terms of y and z:

35. $3^x = 3^y \bullet 3^z$
36. $9^y = \frac{3^z}{3^x}$

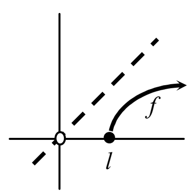
37. $x^3 = y$
38. $3^x = y$
39. $3 \bullet 2^x = 2^y$
40. $\log x^2 = 3\log y$
41. $\log x = 2\log y - \log z$
42. $3\log x = \log y$
43. $\log x = \log y + \log z$
44. $\log\sqrt{x} + \log\sqrt[3]{y} = \log z^2$
45. $\log_7 3 = y;\quad \log_7 2 = z\quad x = \log_3 2$
46. $y = \log_a 9;\ x = \log_a 3$

B. <u>Inverse functions and graphing</u>:
If $y = f(x)$ and $y = g(x)$ are inverse functions, then an ordered pair (a, b) satisfies $y = f(x)$ if and only if the ordered pair (b, a) satisfies $y = g(x)$. In other words, '*f* and *g* are inverses of each other' means $f(a) = b$ if and only if $g(b) = a$

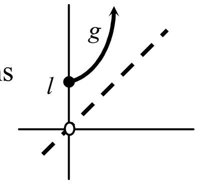
To find the inverse of a function $y = f(x)$
1) Interchange *x* and *y*
2) Solve this equation for *y* in terms of *x*, so $y = g(x)$
3) Then if *g* is a function, *f* and *g* are inverses of each other.

The effect on the graph of $y = f(x)$ when *x* and *y* are switched, is to reflect the graph over the $45°$ line (bisecting quadrants I and III). This reflected graph represents $y = g(x)$.

example: Find the inverse of $f(x) = \sqrt{x-1}$ or $y = \sqrt{x-1}$
1) Switch *x* and *y*:
$x = \sqrt{y-1}$
(note $y \geq 1$ and $x \geq 0$)

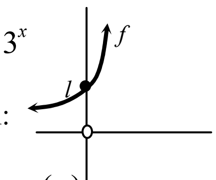


2) Solve for y: $x^2 = y - 1$, so $y = x^2 + 1$ ($x \geq 0$ is still true)
3) Thus $g(x) = x^2 + 1$ (with $x \geq 0$) is the inverse function, and has this graph:

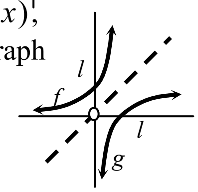


Note that the *f* and *g* graphs are reflections of each other in the $45°$–line, and that the ordered pair (2,5) satisfies *g* and (5,2) satisfies *f*.

example: Find the inverse of $y = f(x) = 3^x$ and graph both functions on one graph:



1) Switch: $x = 3^y$
2) Solve: $\log_3 x = y = g(x)$, the inverse to get the graph of $g(x) = \log_3 x$, reflect the *f* graph over the $45°$–line:



Problems 47-48: Find the inverse function and sketch the graphs of both:

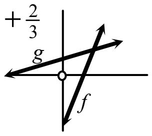
47. $f(x) = 3x - 2$
48. $f(x) = \log_2(-x)$ (note that $-x$ must be positive, which means *x* must be negative)

Problems 49-56: Sketch the graph:

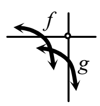
49. $y = x^4$
50. $y = 4^x$
51. $y = 4^{x-1}$
52. $y = \log_4 x$
53. $y = 4^{-x}$
54. $y = -\log_4 x$
55. $y = 4^x - 1$
56. $y = \log_4(x-1)$

<u>Answers</u>:
1. $3^2 = 9,\quad \log_3 9 = 2$
2. $x^3 = x^3, \log_x x^3 = 3$
3. $10^{-2} = 1/100, \log_{10} 1/100 = -2$
4. $169^{1/2} = 13,\ \log_{169} 13 = 1/2$
5. 8
6. $1/9$
7. 1
8. 6
9. 7
10. 8
11. 8
12. 5
13. –5
14. 64
15. 64
16. any real number > 0 and ≠1
17. –1
18. 4
19. 3
20. $1/4$
21. $1/3$
22. $1/3$
23. –2
24. 20
25. $\frac{\log 4 - \log 3}{\log 2}$ (any base; if base$=2$, $x = 2 - \log_2 3$)
26. 1024
27. 10
28. 1
29. $2^{10} = 1024$
30. $1/2$
31. 2
32. $301/477$
33. 50
34. 5
35. $y + z$
36. $z - 2y$
37. $y^{1/3}$
38. $\log_3 y$
39. $y - \log_2 3$
40. $y^{3/2} = y\sqrt{y}$

41. $\frac{y^2}{z}$
42. $\sqrt[3]{y}$
43. yz
44. $\frac{z^4}{\sqrt[3]{y^2}}$
45. $z\!/\!_y$
46. $y\!/\!_2$
47. $g(x)=\frac{1}{3}x+\frac{2}{3}$



48. $g(x)=-2^x$



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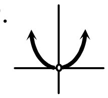

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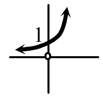

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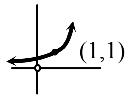

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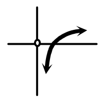

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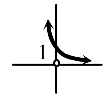

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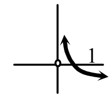

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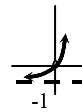

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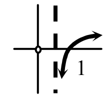
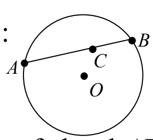


TOPIC 8: MATHEMATICAL MODELING–WORD PROBLEMS

Word Problems:

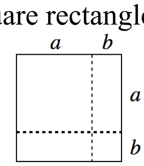
1. $\frac{2}{3}$ of $\frac{1}{6}$ of $\frac{3}{4}$ of a number is 12. What is the number?
2. On the number line, points *P* and *Q* and 2 units apart. *Q* has coordinate *x*. What are the possible coordinates of *P*?
3. What is the number, which when multiplied by 32, gives $32\bullet 46$?
4. If you square a certain number, you get 9^2. What is the number?
5. What is the power of 36 that gives $36^{1/2}$?
6. Point *X* is on each of two given intersecting lines. How many such points *X* are there?
7. Point *Y* is on each of two given circles. How many such points *Y*?
8. Point *Z* is on each of a given circle and a given ellipse. How many such *Z*?
9. Point *R* is on the coordinate plane so its distance from a given point *A* is less than 4. Show in a sketch where *R* could be.

Problems 10-11:



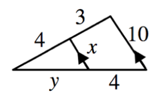
10. If the length of chord *AB* is *x* and length of *CB* is 16, what is *AC*?
11. If $AC = y$ and $CB = z$, how long is *AB* (in terms of *y* and *z*)?

12. This square is cut into two smaller squares and two non-square rectangles as shown.



Before being cut, the large square had area $(a+b)^2$. The two smaller squares have areas a^2 and b^2. Find the total area of the two non-square rectangles. Do the areas of the 4 parts add up to the area of the original square?

13. Find *x* and *y*:



14. When constructing an equilateral triangle with an area that is 100 times the area of a given equilateral triangle, what length should be used for a side?

Problems 15-16: *x* and *y* are numbers, and two *x*'s equal three *y*'s:

15. Which of *x* or *y* must be larger?
16. What is the ratio of *x* to *y*?

Problems 17-21: A plane has a certain speed in still air. In still air, it goes 1350 miles in 3 hours:

17. What is its (still air) speed?
18. How long does it take to fly 2000 miles?
19. How far does the plane go in *x* hours?
20. If the plane flies against a 50 mph headwind, what is its ground speed?
21. If it has fuel for 7.5 hours of flying time, how far can it go against this headwind?

Problems 22-25: Georgie and Porgie bake pies. Georgie can complete 30 pies an hour.

22. How many can he make in one minute?
23. How many can he make in 10 minutes?
24. How many can he make in *x* minutes?
25. How long does he take to make 200 pies?

Problems 26-28: Porgie can finish 45 pies an hour:

26. How many can she make in one minute?
27. How many can she make in 20 minutes?
28. How many can she make in x minutes?

Problems 29-32: If they work together, how many pies can they produce in:

29. 1 minute	31. 80 minutes
30. x minutes	32. 3 hours

Problems 33-41: A nurse needs to mix some alcohol solutions, given as a percent by weight of alcohol in water. Thus in a 3% solution, 3% of the weight would be alcohol. She mixes x grams of 3% solution, y grams of 10% solution, and 10 grams of pure water to get a total of 140 grams of a solution which is 8% alcohol:

33. In terms of x, how many grams of alcohol are in the 3% solution?
34. The y grams of 10% solution would include how many grams of alcohol?
35. How many grams of solution are in the final mix (the 8% solution)?
36. Write an expression in terms of x and y for the total number of grams in the 8% solution contributed by the three ingredients (the 3%, 10%, and water).
37. Use your last two answers to write a 'total grams equation'.
38. How many grams of alcohol are in the 8%.
39. Write an expression in terms of x and y for the total number of grams of alcohol in the final solution.
40. Use the last two answers to write a 'total grams of alcohol equation'.
41. How many grams of each solution are needed?

42. Half the square of a number is 18. What is the number?
43. If the square of twice a number is 81, what is the number?
44. Given a positive number x. The square of a positive number y is at least 4 times x. How small can y be?
45. Twice the square of half of a number is x. What is the number?

Problems 46-48: Half of x is the same as one-third of y.

46. Which of x and y is the larger?
47. Write the ratio $x: y$ as the ratio of two integers.
48. How many x's equal 30 y's?

Problems 49-50: A gathering has twice as many women as men. If W is the number of women and M is the number of men…

49. Which is correct: $2M=W$ or $M=2W$
50. Write the ratio $\frac{W}{M+W}$ as the ratio of two integers.

Problems 51-53: If A is increased by 25%, it equals B.

51. Which is larger, B or the original A?
52. B is what percent of A?
53. A is what percent of B?

Problems 54-56: If C is decreased by 40%, it equals D.

54. Which is larger, D or the original C?
55. C is what percent of D?
56. D is what percent of C?

Problems 57-58: The length of a rectangle is increased by 25% and its width is decreased by 40%.

57. Its new area is what percent of its old area?
58. By what percent has the old area increased or decreased?

Problems 59-61: Your wage is increased by 20%, then the new amount is cut by 20% (of the new amount).

59. Will this result in a wage, which is higher than, lower than, or the same as the original wage?
60. What percent of the original wage is this final wage?
61. If the above steps were reversed (20% cut followed by 20% increase), the final wage would be what percent of the original wage?

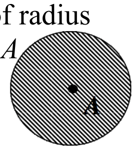
Problems 62-75: Write an equation for each of the following statement about real numbers and tell whether it is true or false:

62. The product of the squares of two numbers is the square of the product of the two numbers.
63. The square of the sum of two numbers is the sum of the squares of the two numbers
64. The square of the square root of a number is the square root of the square of the number.
65. The square root of the sum of the squares of two numbers is the sum of the two numbers.
66. The sum of the absolute vales of two numbers is the absolute value of the sum of the two numbers.

67. The product of the absolute values of two numbers is the absolute value of the product of the two numbers.
68. The negative of the absolute value of a number is the absolute value of the negative of the number.
69. The cube root of the square of a number is the cube of the square root of the number.
70. The reciprocal of the negative of a number is the negative of the reciprocal of the number.
71. The reciprocal of the sum of two numbers is the sum of the reciprocals of the two numbers.
72. The reciprocal of the product of two numbers is the product of the reciprocals of the two numbers.
73. The reciprocal of the quotient of two numbers is the quotient of the reciprocals of the two numbers.
74. The multiple of x which gives xy is the multiple of y which gives yx.
75. The power of 2, which gives 2^{xy} is the power of 2, which gives 2^{yx}.

Answers:

1. 144
2. $x+2,\ x-2$
3. 46
4. 9
5. $\frac{1}{2}$
6. 1
7. 0, 1, or 2
8. 0, 1, 2, 3, or 4
9. Inside circle of radius 4 centered on A



10. $x-16$
11. $y+z$
12. $2ab$, yes: $(a+b)^2 = a^2+2ab+b^2$
13. $x=\frac{40}{7};\ \ y=\frac{16}{3}$
14. 10 times the original side
15. x
16. $\frac{3}{2}$
17. 450 mph
18. $4\frac{4}{9}$ hours
19. $450x$ miles
20. 400 mph
21. 3000 miles
22. $\frac{1}{2}$
23. 5
24. $\frac{x}{2}$
25. 400 min.
26. $\frac{3}{4}$
27. 15
28. $\frac{3x}{4}$
29. $\frac{5}{4}$
30. $\frac{5x}{4}$
31. 100
32. 225
33. $.03x$
34. $.1y$
35. 140
36. $x+y+10$
37. $x+y+10=140$
38. 11.2
39. $.03x+.1y$
40. $.03x+.1y=11.2$
41. $x=\frac{180}{7};\ \ y=\frac{730}{7}$
42. 6, -6
43. 4.5, -4.5
44. $2\sqrt{x}$
45. $\sqrt{2x}$
46. y
47. $2:3$
48. 45
49. $2M=W$
50. $\frac{2}{3}$
51. B
52. 125%
53. 80%
54. C
55. $166\frac{2}{3}$%
56. 60%
57. 75%
58. 25% decrease
59. lower
60. 96%
61. same (96%)
62. $a^2b^2=(ab)^2$ T
63. $(a+b)^2=a^2+b^2$ F
64. $\left(\sqrt{a}\right)^2=\sqrt{a^2}$ T (if all expressions are real)
65. $\sqrt{a^2+b^2}=a+b$ F
66. $|a|+|b|=|a+b|$ F
67. $|a|\bullet|b|=|ab|$ T
68. $-|a|=|-a|$ F
69. $\sqrt[3]{a^2}=\left(\sqrt{a}\right)^3$ F
70. $\frac{1}{-a}=-\frac{1}{a}$ T
71. $\frac{1}{a+b}=\frac{1}{a}+\frac{1}{b}$ F
72. $\frac{1}{ab}=\frac{1}{a}\bullet\frac{1}{b}$ T
73. $\frac{\frac{1}{a}}{\frac{1}{b}}=\frac{1}{\frac{a}{b}}$ T
74. $y=x$ F
75. $xy=yx$ T