INTERMEDIATE ALGEBRA READINESS DIAGNOSTIC TEST PRACTICE

Directions: Study the examples, work the problems, then check your answers at the end of each topic. If you don't get the answer given, check your work and look for mistakes. If you have trouble, ask a math teacher or someone else who understands this topic.

TOPIC 1: ELEMENTARY OPERATIONS

A. Algebraic operations, grouping, evaluation:
To evaluate an expression, first calculate the powers, then multiply and divide in order from left to right, and finally add and subtract in order from left to right. Parentheses have preference.

example: $14 - 3^2 = 14 - 9 = 5$
example: $2 \bullet 4 + 3 \bullet 5 = 8 + 15 = 23$
example: $10 - 2 \bullet 3^2 = 10 - 2 \bullet 9 = 10 - 18 = -8$
example: $(10 - 2) \bullet 3^2 = 8 \bullet 9 = 72$

Problems 1-7: Find the value:

1. $2^3 =$
2. $-2^4 =$
3. $4 + 2 \bullet 5 =$
4. $3^2 - 2 \bullet 3 + 1 =$
5. $0^4 =$
6. $(-2)^4 =$
7. $1^5 =$

Problems 8-13: Find the value if $a = -3$, $b = 2$, $c = 0, d = 1$, and $e = -3$:

8. $a - e =$
9. $e^2 + (d - ab)c =$
10. $a - (bc - d) + e =$
11. $\frac{e}{d} + \frac{b}{a} - \frac{2d}{e} =$
12. $\frac{b}{e} =$
13. $\frac{d}{c} =$

Combine like terms when possible:

example: $3x + y^2 - (x + 2y^2)$
$= 3x - x + y^2 - 2y^2 = 2x - y^2$
example: $a - a^2 + a = 2a - a^2$

Problems 14-20: Simplify:

14. $6x + 3 - x - 7 =$
15. $2(3 - t) =$
16. $10r - 5(2r - 3y) =$
17. $x^2 - (x - x^2) =$
18. $3a - 2\big(4(a - 2b) - 3a\big) =$
19. $3(a + b) - 2(a - b) =$
20. $1 + x - 2x + 3x - 4x =$

B. Simplifying fractional expressions:

example: $\frac{27}{36} = \frac{9\bullet 3}{9\bullet 4} = \frac{9}{9} \bullet \frac{3}{4} = 1 \bullet \frac{3}{4} = 3/4$
(note that you must be able to find a common factor - in this case 9 - in both the top and bottom in order to reduce a fraction.)
example: $\frac{3a}{12ab} = \frac{3a \bullet 1}{3a \bullet 4b} = \frac{3a}{3a} \bullet \frac{1}{4b} = 1 \bullet \frac{1}{4b} = \frac{1}{4b}$
(common factor: $3a$)

Problems 21-32: Reduce:

21. $\frac{13}{52} =$
22. $\frac{26}{65} =$
23. $\frac{3+6}{3+9} =$
24. $\frac{6axy}{15by} =$
25. $\frac{19a^2}{95a} =$
26. $\frac{14x - 7y}{7y} =$
27. $\frac{5a+b}{5a+c} =$
28. $\frac{x-4}{4-x} =$
29. $\frac{2(x+4)(x-5)}{(x-5)(x-4)} =$
30. $\frac{x^2 - 9x}{x - 9} =$
31. $\frac{8(x-1)^2}{6(x^2-1)} =$
32. $\frac{2x^2 - x - 1}{x^2 - 2x + 1} =$

example: $\frac{3}{x} \bullet \frac{y}{15} \bullet \frac{10x}{y^2} = \frac{3 \bullet 10 \bullet x \bullet y}{15 \bullet x \bullet y^2} =$
$\frac{3}{3} \bullet \frac{5}{5} \bullet \frac{2}{1} \bullet \frac{x}{x} \bullet \frac{y}{y} \bullet \frac{1}{y} =$
$1 \bullet 1 \bullet 2 \bullet 1 \bullet 1 \bullet \frac{1}{y} = \frac{2}{y}$

Problems 33-34: Simplify:

33. $\frac{4x}{6} \bullet \frac{xy}{y^2} \bullet \frac{3y}{2} =$
34. $\frac{x^2 - 3x}{x - 4} \bullet \frac{x(x-4)}{2x - 6} =$

C. Laws of integer exponents:

I. $a^b \bullet a^c = a^{b+c}$
II. $\frac{a^b}{a^c} = a^{b-c}$
III. $(a^b)^c = a^{bc}$
IV. $(ab)^c = a^c \bullet b^c$
V. $\left(\frac{a}{b}\right)^c = \frac{a^c}{b^c}$
VI. $a^0 = 1$ (if $a \neq 0$)
VII. $a^{-b} = \frac{1}{a^b}$

Problems 35-44: Find x:

35. $2^3 \bullet 2^4 = 2^x$
36. $\frac{2^3}{2^4} = 2^x$
37. $3^{-4} = \frac{1}{3^x}$
38. $\frac{5^2}{5^2} = 5^x$
39. $\left(2^4\right)^3 = 2^x$
40. $8 = 2^x$
41. $a^x = a^3 \bullet a$
42. $\frac{b^{10}}{b^5} = b^x$
43. $\frac{1}{c^{-4}} = c^x$
44. $\frac{a^{3y-2}}{a^{2y-3}} = a^x$

Problems 45-59: Simplify:

45. $8x^0 =$
46. $3^{-4} =$
47. $2^3 \bullet 2^4 =$
48. $0^5 =$
49. $5^0 =$
50. $(-3)^3 - 3^3 =$
51. $2^x \bullet 4^{x-1} =$
52. $\frac{2^{c+3}}{2^{c-3}} =$
53. $2^{c+3} \bullet 2^{c-3} =$
54. $\frac{8^x}{2^{x-1}} =$

55. $\frac{2x^{-3}}{6x^{-4}} =$

56. $\left(a^{x+3}\right)^{x} =$

57. $\frac{a^{3x-2}}{a^{2x-3}} =$

58. $\left(-2a^{2}\right)^{4}\left(ab^{2}\right) =$

59. $2\left(4xy^{2}\right)^{-1}\left(-2x^{-1}y\right)^{2} =$

D. Scientific notation:

example: $32800 = 3.2800\times10^{4}$ if the zeros in the ten's and one's places are significant. If the one's zero is not, write 3.280×10^{4}; if neither is significant: 3.28×10^{4}

example: $.004031 = 4.031\times10^{-3}$

example: $2\times10^{2} = 200$

example: $9.9\times10^{-1} = .99$

Note that scientific form always looks like $a\times10^{n}$ where $1 \le a < 10$, and n is an integer power of 10.

Problems 60-63: Write in scientific notation:

60. $93{,}000{,}000 =$

61. $.000042 =$

62. $5.07 =$

63. $-32 =$

Problems 64-66: Write in standard notation:

64. $1.4030\times10^{3} =$

65. $-9.11\times10^{-2} =$

66. $4\times10^{-6} =$

To compute with numbers written in scientific form, separate the parts, compute, and then recombine.

example: $\left(3.14\times10^{5}\right)(2) = (3.14)(2)\times10^{5}$
$= 6.28\times10^{5}$

example: $\frac{4.28\times10^{6}}{2.14\times10^{-2}} = \frac{4.28}{2.14}\times\frac{10^{6}}{10^{-2}} = 2.00\times10^{8}$

example: $\frac{2.01\times10^{-3}}{8.04\times10^{-6}} = .250\times10^{3} = 2.50\times10^{2}$

Problems 67-74: Write answer in scientific notation:

67. $10^{40}\times10^{-2} =$

68. $\frac{10^{-40}}{10^{-10}} =$

69. $\frac{1.86\times10^{4}}{3\times10^{-1}} =$

70. $\frac{3.6\times10^{-5}}{1.8\times10^{-8}} =$

71. $\frac{1.8\times10^{-8}}{3.6\times10^{-5}} =$

72. $\left(4\times10^{-3}\right)^{2} =$

73. $\left(2.5\times10^{2}\right)^{-1} =$

74. $\frac{\left(-2.92\times10^{3}\right)\left(4.1\times10^{7}\right)}{-8.2\times10^{-3}} =$

E. Absolute value:

example: $|3| = 3$

example: $|-3| = 3$

example: $|a|$ depends on a

if $a \ge 0$, $|a| = a$

if $a < 0$, $|a| = -a$

example: $-|-3| = -3$

Problems 75-78: Find the value:

75. $|0| =$

76. $\frac{|a|}{a} =$

77. $|3| + |-3| =$

78. $|3| - |-3| =$

Problems 79-84: If $x = -4$, find:

79. $|x+1| =$

80. $|1-x| =$

81. $-|x| =$

82. $x + |x| =$

83. $|-3x| =$

84. $\left|\,(x - |(x - |x|\,)|\,)\right| =$

Answers:

1. 8
2. –16
3. 14
4. 4
5. 0
6. 16
7. 1
8. 0
9. 9
10. –5
11. –3
12. $-\frac{2}{3}$
13. no value (undefined)
14. $5x-4$
15. $6-2t$
16. $15y$
17. $2x^{2}-x$
18. $a+16b$
19. $a+5b$
20. $1-2x$
21. $\frac{1}{4}$
22. $\frac{2}{5}$
23. $\frac{3}{4}$
24. $\frac{2ax}{5b}$
25. $\frac{a}{5}$
26. $\frac{2x-y}{y}$
27. $\frac{5a+b}{5a+c}$
28. -1
29. $\frac{2(x+4)}{x-4}$
30. x
31. $\frac{4(x-1)}{3(x+1)}$
32. $\frac{2x+1}{x-1}$
33. x^{2}
34. $\frac{x^{2}}{2}$
35. 7
36. –1
37. 4
38. 0
39. 12
40. 3
41. 4
42. 5
43. 4
44. $y+1$
45. 8
46. $\frac{1}{81}$
47. 128
48. 0
49. 1
50. –54
51. 2^{3x-2}
52. 64
53. 4^{c}
54. 2^{2x+1}
55. $\frac{x}{3}$

56. a^{x^2+3x}
57. a^{x+1}
58. $16a^9b^2$
59. $2/x^3$
60. 9.3×10^7
61. 4.2×10^{-5}
62. 5.07
63. -3.2×10
64. 1403.0
65. -.0911
66. .000004
67. 1×10^{38}
68. 1×10^{-30}
69. 6.2×10^4
70. 2.0×10^3
71. 5.0×10^{-4}
72. 1.6×10^{-5}
73. 4.0×10^{-3}
74. 1.46×10^{13}
75. 0
76. 1 if $a>0$;
    -1 if $a<0$;
    (no value if $a=0$)
77. 6
78. 0
79. 3
80. 5
81. –4
82. 0
83. 12
84. 12

TOPIC 2: RATIONAL EXPRESSIONS

A. <u>Adding and subtracting fractions</u>:
If denominators are the same, combine the numerators:

example: $\frac{3x}{y}-\frac{x}{y}=\frac{3x-x}{y}=\frac{2x}{y}$

Problems 1-5: Find the sum or difference as indicated (reduce if possible):

1. $\frac{4}{7}+\frac{2}{7}=$
2. $\frac{3}{x-3}-\frac{x}{x-3}=$
3. $\frac{b-a}{b+a}-\frac{a-b}{b+a}=$
4. $\frac{x+2}{x^2+2x}-\frac{3y^2}{xy^2}=$
5. $\frac{3a}{b}+\frac{2}{b}-\frac{a}{b}=$

If denominators are different, find equivalent fractions with common denominators:

example: $3/4$ is equivalent to how many eighths?
$\frac{3}{4}=\frac{}{8}$; $\frac{3}{4}=1\bullet\frac{3}{4}=\frac{2}{2}\bullet\frac{3}{4}=\frac{2\bullet3}{2\bullet4}=\frac{6}{8}$

example: $\frac{6}{5a}=\frac{}{5ab}$; $\frac{6}{5a}=\frac{b}{b}\bullet\frac{6}{5a}=\frac{6b}{5ab}$

example: $\frac{3x+2}{x+1}=\frac{}{4(x+1)}$; $\frac{3x+2}{x+1}=\frac{4}{4}\bullet\frac{3x+2}{x+1}=\frac{12x+8}{4x+4}$

example: $\frac{x-1}{x+1}=\frac{}{(x+1)(x-2)}$;
$\frac{x-1}{x+1}=\frac{(x-2)(x-1)}{(x-2)(x+1)}=\frac{x^2-3x+2}{(x+1)(x-2)}$

Problems 6-10: Complete:

6. $\frac{4}{9}=\frac{}{72}$
7. $\frac{3x}{7}=\frac{}{7y}$
8. $\frac{x+3}{x+2}=\frac{}{(x-1)(x+2)}$
9. $\frac{30-15a}{15-15b}=\frac{}{(1+b)(1-b)}$
10. $\frac{x-6}{6-x}=\frac{}{-2}$

How to get the lowest common denominator (LCD) by finding the least common multiple (LCM) of all denominators:

example: $5/6$ and $8/15$.
First find LCM of 6 and 15:
$6=2\bullet3$
$15=3\bullet5$
$\text{LCM}=2\bullet3\bullet5=30$
so, $\frac{5}{6}=\frac{25}{30}$, and $\frac{8}{15}=\frac{16}{30}$

example: $3/4$ and $1/6a$:
$4=2\bullet2$
$6a=2\bullet3\bullet a$
$\text{LCM}=2\bullet2\bullet3\bullet a=12a$
so, $\frac{3}{4}=\frac{9a}{12a}$, and $\frac{1}{6a}=\frac{2}{12a}$

example: $\frac{2}{3(x+2)}$ and $\frac{ax}{6(x+1)}$
$3(x+2)=3\bullet(x+2)$
$6(x+1)=2\bullet3\bullet(x+1)$
$\text{LCM}=2\bullet3\bullet(x+1)\bullet(x+2)$
so, $\frac{2}{3(x+2)}=\frac{2\bullet2(x+1)}{2\bullet3(x+1)(x+2)}=\frac{4(x+1)}{6(x+1)(x+2)}$ and
$\frac{ax}{6(x+1)}=\frac{ax(x+2)}{6(x+1)(x+2)}$

Problems 11-16: Find equivalent fractions with the lowest common denominator:

11. $\frac{2}{3}$ and $\frac{2}{9}$
12. $\frac{3}{x}$ and 5
13. $\frac{x}{3}$ and $\frac{-4}{x+1}$
14. $\frac{3}{x-2}$ and $\frac{4}{2-x}$
15. $\frac{x}{15(x^2-2)}$ and $\frac{7x(y-1)}{10(x-1)}$
16. $\frac{1}{x}$, $\frac{3x}{x+1}$, and $\frac{x^2}{x^2+x}$

After finding equivalent fractions with common denominators, proceed as before (combine numerators):

example: $\frac{a}{2}-\frac{a}{4}=\frac{2a}{4}-\frac{a}{4}=\frac{2a-a}{4}=\frac{a}{4}$

example: $\frac{3}{x-1}+\frac{1}{x+2}$
$=\frac{3(x+2)}{(x-1)(x+2)}+\frac{(x-1)}{(x-1)(x+2)}$
$=\frac{3x+6+x-1}{(x-1)(x+2)}=\frac{4x+5}{(x-1)(x+2)}$

Problems 17-30: Find the sum or difference:

17. $\frac{3}{a}-\frac{1}{2a}=$
18. $\frac{3}{x}-\frac{2}{a}=$
19. $\frac{4}{5}-\frac{2}{x}=$
20. $\frac{2}{5}+2=$
21. $\frac{a}{b}-2=$
22. $a-\frac{c}{b}=$
23. $\frac{1}{a}+\frac{1}{b}=$
24. $a-\frac{1}{a}=$
25. $\frac{x}{x-1}+\frac{x}{1-x}=$
26. $\frac{3x-2}{x-2}-\frac{2}{x+2}=$
27. $\frac{2x-1}{x+1}-\frac{2x-1}{x-2}=$

28. $\frac{1}{(x-1)(x-2)}+\frac{1}{(x-2)(x-3)}-\frac{2}{(x-3)(x-1)}=$

29. $\frac{x}{x-2}-\frac{4}{x^2-2x}=$

30. $\frac{x}{x-2}-\frac{4}{x^2-4}=$

B. <u>Multiplying fractions</u>:
Multiply the top numbers, multiply the bottom numbers, and reduce if possible.

example: $\frac{3}{4}\bullet\frac{2}{5}=\frac{6}{20}=\frac{3}{10}$

example: $\frac{3(x+1)}{x-2}\bullet\frac{x^2-4}{x^2-1}=\frac{3(x+1)(x+2)(x-2)}{(x-2)(x+1)(x-1)}=\frac{3x+6}{x-1}$

31. $\frac{2}{3}\bullet\frac{3}{8}=$

32. $\frac{a}{b}\bullet\frac{c}{d}=$

33. $\frac{2}{7a}\bullet\frac{ab}{12}=$

34. $\frac{3(x+4)}{5y}\bullet\frac{5y^3}{x^2-16}=$

35. $\frac{(a+b)^3}{(x-y)^2}\bullet\frac{(x-y)}{(5-p)}\bullet\frac{(p-5)^2}{(a+b)^2}=$

36. $\left(\frac{3}{4}\right)^2=$

37. $\left(\frac{2a^3}{5b}\right)^3=$

38. $\left(2\frac{1}{2}\right)^2=$

C. <u>Dividing fractions</u>:
Make a compound fraction and then multiply the top and bottom (of the big fraction) by the LCD of both:

example: $\frac{a}{b}\div\frac{c}{d}=\frac{\frac{a}{b}}{\frac{c}{d}}=\frac{\frac{a}{b}\bullet bd}{\frac{c}{d}\bullet bd}=\frac{ad}{bc}$

example: $\frac{7}{\frac{2}{3}-\frac{1}{2}}=\frac{7\bullet 6}{\left(\frac{2}{3}-\frac{1}{2}\right)\bullet 6}=\frac{42}{4-3}=42/1=42$

example: $\frac{5x}{2y}\div 2x=\frac{\frac{5x}{2y}}{2x}=\frac{\frac{5x}{2y}\bullet 2y}{2x\bullet 2y}=\frac{5x}{4xy}=\frac{5}{4y}$

39. $\frac{3}{4}\div\frac{2}{3}=$

40. $11\frac{3}{8}\div\frac{3}{4}=$

41. $\frac{3}{4}\div 2=$

42. $\frac{a}{b}\div 3=$

43. $\frac{3}{a}\div\frac{b}{3}=$

44. $\frac{\frac{x+7}{x^2-9}}{\frac{1}{x-3}}=$

45. $\frac{a-4}{\frac{3}{a}-2}=$

46. $\frac{2a-b}{\frac{1}{2}}=$

47. $\frac{2}{\frac{3}{4}}=$

48. $\frac{\frac{2}{3}}{4}=$

49. $\frac{\frac{a}{b}}{c}=$

50. $\frac{a}{\frac{b}{c}}=$

51. $\frac{\frac{1}{a}-\frac{1}{b}}{\frac{1}{a}+\frac{1}{b}}=$

52. $\frac{\frac{1}{2a}-\frac{1}{b}}{\frac{1}{a}-\frac{1}{2b}}=$

53. $\frac{\frac{1}{a}-\frac{1}{b}}{\frac{1}{ab}}=$

<u>Answers</u>:

1. $6/7$
2. -1
3. $\frac{2b-2a}{b+a}$
4. $-2/x$
5. $\frac{2a+2}{b}$
6. 32
7. $3xy$
8. x^2+2x-3
9. $2+2b-a-ab$
10. 2
11. $\frac{6}{9}$, $\frac{2}{9}$
12. $\frac{3}{x}$, $\frac{5x}{x}$
13. $\frac{x(x+1)}{3(x+1)}$, $\frac{-12}{3(x+1)}$
14. $\frac{3}{x-2}$, $\frac{-4}{x-2}$
15. $\frac{2x(x-1)}{30(x^2-2)(x-1)}$, $\frac{21x(y-1)(x^2-2)}{30(x^2-2)(x-1)}$
16. $\frac{x+1}{x(x+1)}$, $\frac{3x^2}{x(x+1)}$, $\frac{x^2}{x(x+1)}$
17. $\frac{5}{2a}$
18. $\frac{3a-2x}{ax}$
19. $\frac{4x-10}{5x}$
20. $12/5$
21. $\frac{a-2b}{b}$
22. $\frac{ab-c}{b}$
23. $\frac{a+b}{ab}$
24. $\frac{a^2-1}{a}$
25. 0
26. $\frac{3x^2+2x}{x^2-4}$
27. $\frac{-3(2x-1)}{(x+1)(x-2)}$
28. 0
29. $\frac{x+2}{x}$
30. $\frac{x^2+2x-4}{x^2-4}$
31. $1/4$
32. $\frac{ac}{bd}$
33. $\frac{b}{42}$
34. $\frac{3y^2}{x-4}$
35. $\frac{(a+b)(5-p)}{x-y}$
36. $9/16$
37. $\frac{8a^9}{125b^3}$
38. $25/4$
39. $9/8$
40. $91/6$
41. $3/8$
42. $\frac{a}{3b}$
43. $\frac{9}{ab}$
44. $\frac{x+7}{x+3}$
45. $\frac{a^2-4a}{3-2a}$
46. $4a-2b$
47. $8/3$
48. $1/6$
49. $\frac{a}{bc}$
50. $\frac{ac}{b}$
51. $\frac{b-a}{b+a}$
52. $\frac{b-2a}{2b-a}$
53. $b-a$

TOPIC 3: EXPONENTS and RADICALS

A. <u>Definitions of powers and roots</u>:

Problems 1-20: Find the value:

1. $2^3 =$
2. $3^2 =$
3. $-4^2 =$
4. $(-4)^2 =$
5. $0^4 =$
6. $1^4 =$
7. $\sqrt{64} =$
8. $\sqrt[3]{64} =$
9. $\sqrt[6]{64} =$
10. $-\sqrt{49} =$
11. $\sqrt[3]{-125} =$
12. $\sqrt{5^2} =$
13. $\sqrt{(-5)^2} =$
14. $\sqrt{x^2} =$
15. $\sqrt[3]{a^3} =$
16. $\sqrt{\frac{1}{4}} =$
17. $\sqrt{.04} =$
18. $\left(\frac{2}{3}\right)^4 =$
19. $\sqrt[4]{81a^8} =$
20. $\sqrt{7} \bullet \sqrt{7} =$

B. <u>Laws of integer exponents</u>:

I. $a^b \bullet a^c = a^{b+c}$
II. $\frac{a^b}{a^c} = a^{b-c}$
III. $\left(a^b\right)^c = a^{bc}$
IV. $(ab)^c = a^c \bullet b^c$
V. $\left(\frac{a}{b}\right)^c = \frac{a^c}{b^c}$
VI. $a^0 = 1$ (if $a \neq 0$)
VII. $a^{-b} = \frac{1}{a^b}$

Problems 21-30: Find *x*:

21. $2^3 \bullet 2^4 = 2^x$
22. $\frac{2^3}{2^4} = 2^x$
23. $3^{-4} = \frac{1}{3^x}$
24. $\frac{5^2}{5^2} = 5^x$
25. $\left(2^3\right)^4 = 2^x$
26. $8 = 2^x$
27. $a^3 \bullet a = a^x$
28. $\frac{b^{10}}{b^5} = b^x$
29. $\frac{1}{c^{-4}} = c^x$
30. $\frac{a^{3y-2}}{a^{2y-3}} = a^x$

Problems 31-43: Find the value:

31. $7x^0 =$
32. $3^{-4} =$
33. $2^3 \bullet 2^4 =$
34. $0^5 =$
35. $5^0 =$
36. $(-3)^3 - 3^3 =$
37. $x^{c+3} \bullet x^{c-3} =$
38. $2^x \bullet 4^{x-1} =$
39. $\frac{x^{c+3}}{x^{c-3}} =$
40. $\frac{8^x}{2^{x-1}} =$
41. $\frac{2x^{-3}}{6x^{-4}} =$
42. $\left(a^{x+3}\right)^{x-3} =$
43. $\frac{a^{3x-2}}{a^{2x-3}} =$

Problems 44-47: Write given two ways:

Given	No negative powers	No fraction
44. $\frac{d^{-4}}{d^4}$		
45. $\left(\frac{3x^3}{y}\right)^{-2}$		
46. $\left(\frac{a^2bc}{2ab^2c}\right)^3$		
47. $\frac{x^2y^3z^{-1}}{x^5y^{-6}z^{-3}}$		

C. <u>Laws of rational exponents, and radicals</u>:

Assume all radicals are real numbers:

I. If *r* is a positive integer, *p* is an integer, and $a \geq 0$, then $a^{p/r} = \sqrt[r]{a^p} = \left(\sqrt[r]{a}\right)^p$ which is a real number. (Also true if *r* is a positive odd integer and $a < 0$)
Think of $\frac{p}{r}$ as $\frac{\text{power}}{\text{root}}$

II. $\sqrt[r]{ab} = \sqrt[r]{a} \bullet \sqrt[r]{b}$, or $(ab)^{1/r} = a^{1/r} \bullet b^{1/r}$

III. $\sqrt[r]{\frac{a}{b}} = \frac{\sqrt[r]{a}}{\sqrt[r]{b}}$, or $\left(a/b\right)^{1/r} = \frac{a^{1/r}}{b^{1/r}}$

IV. $\sqrt[rs]{a} = \sqrt[r]{\sqrt[s]{a}} = \sqrt[s]{\sqrt[r]{a}}$
or $a^{1/rs} = \left(a^{1/s}\right)^{1/r} = \left(a^{1/r}\right)^{1/s}$

Problems 48-53: Write as a radical:

48. $3^{1/2} =$
49. $4^{2/3} =$
50. $\left(1/2\right)^{1/3} =$
51. $x^{3/2} =$
52. $2x^{1/2} =$
53. $(2x)^{1/2} =$

Problems 54-57: Write as a fractional power:

54. $\sqrt{5} =$
55. $\sqrt{2^3} =$
56. $\sqrt[3]{a} =$
57. $\frac{1}{\sqrt{a}} =$

Problems 58-62: Find *x*:

58. $\sqrt{4} \bullet \sqrt{9} = \sqrt{x}$
59. $\sqrt{x} = \frac{\sqrt{4}}{\sqrt{9}}$
60. $\sqrt{\sqrt[3]{64}} = \sqrt{x}$
61. $\sqrt[3]{\sqrt{64}} = x$
62. $x = \frac{8^{2/3}}{4^{3/2}}$

Problems 63-64: Write with positive exponents:

63. $\left(9x^6y^{-2}\right)^{1/2} =$
64. $\left(-8a^6b^{-12}\right)^{-2/3} =$

D. <u>Simplification of radicals</u>:

example: $\sqrt{32} = \sqrt{16} \bullet \sqrt{2} = 4\sqrt{2}$
example: $\sqrt[3]{72} = \sqrt[3]{8} \bullet \sqrt[3]{9} = 2\sqrt[3]{9}$

example: $\sqrt[3]{54}+4\sqrt[3]{16}=$
$\sqrt[3]{27}\bullet\sqrt[3]{2}+4\sqrt[3]{8}\bullet\sqrt[3]{2}=$
$3\sqrt[3]{2}+4\bullet 2\sqrt[3]{2}=$
$3\sqrt[3]{2}+8\sqrt[3]{2}=11\sqrt[3]{2}$

example: $\sqrt{8}-\sqrt{2}=2\sqrt{2}-\sqrt{2}=\sqrt{2}$

Problems 65-82: Simplify (assume all radicals are real numbers):

65. $-\sqrt{81}=$
66. $\sqrt{50}=$
67. $3\sqrt{12}=$
68. $\sqrt[3]{54}=$
69. $\sqrt{52}=$
70. $2\sqrt{3}+\sqrt{27}-\sqrt{75}=$
71. $\sqrt{x^5}=$
72. $\sqrt{4x^6}=$
73. $x\sqrt{2x}+2\sqrt{2x^3}+\frac{2x^2}{\sqrt{2x}}=$
74. $\sqrt{a^2}=$
75. $\sqrt{a^3}=$
76. $\sqrt[3]{a^5}=$
77. $3\sqrt{2}+\sqrt{2}=$
78. $5\sqrt{3}-\sqrt{3}=$
79. $\sqrt{9x^2-9y^2}=$
80. $\sqrt{9x^2+9y^2}=$
81. $\sqrt{9(x+y)^2}=$
82. $\sqrt[3]{64(x+y)^3}=$

E. <u>Rationalization of denominators</u>:

example: $\sqrt{\frac{5}{8}}=\frac{\sqrt{5}}{\sqrt{8}}=\frac{\sqrt{5}}{\sqrt{8}}\bullet\frac{\sqrt{2}}{\sqrt{2}}=\frac{\sqrt{10}}{\sqrt{16}}=\frac{\sqrt{10}}{4}$

example: $\frac{1}{\sqrt[3]{2}}=\frac{1}{\sqrt[3]{2}}\bullet\frac{\sqrt[3]{4}}{\sqrt[3]{4}}=\frac{\sqrt[3]{4}}{\sqrt[3]{8}}=\frac{\sqrt[3]{4}}{2}$

example: $\frac{\sqrt{3}}{\sqrt{3}-1}=\frac{\sqrt{3}}{\sqrt{3}-1}\bullet\frac{\sqrt{3}+1}{\sqrt{3}+1}=\frac{\sqrt{9}+\sqrt{3}}{3-1}=\frac{3+\sqrt{3}}{2}$

Problems 83-91: Simplify:

83. $\sqrt{\frac{2}{3}}=$
84. $\frac{1}{\sqrt{5}}=$
85. $\frac{3}{\sqrt{3}}=$
86. $\frac{\sqrt{a}}{\sqrt{b}}=$
87. $\sqrt[3]{\frac{2}{3}}=$
88. $\sqrt{2}+\frac{1}{\sqrt{2}}=$
89. $\frac{3}{\sqrt{2}+1}=$
90. $\frac{\sqrt{3}}{1-\sqrt{3}}=$
91. $\frac{\sqrt{3}+2}{\sqrt{3}-2}=$

<u>Answers</u>:

1. 8
2. 9
3. –16
4. 16
5. 0
6. 1
7. 8
8. 4
9. 2
10. –7
11. –5
12. 5
13. 5
14. x if $x\ge 0$; $-x$ if $x<0$
15. a
16. $\frac{1}{2}$
17. 0.2
18. $\frac{16}{81}$
19. $3a^2$
20. 7
21. 7
22. –1
23. 4
24. 0
25. 12
26. 3
27. 4
28. 5
29. 4
30. $y+1$
31. 7
32. $\frac{1}{81}$
33. 128
34. 0
35. 1
36. –54
37. x^{2c}
38. 2^{3x-2}
39. x^6
40. 2^{2x+1}
41. $\frac{x}{3}$
42. a^{x^2-9}
43. a^{x+1}
44. $\frac{1}{d^8}=d^{-8}$
45. $\frac{y^2}{9x^6}=9^{-1}x^{-6}y^2$
46. $\frac{a^3}{8b^3}=8^{-1}a^3b^{-3}$
47. $\frac{y^9z^2}{x^3}=x^{-3}y^9z^2$
48. $\sqrt{3}$
49. $\sqrt[3]{16}$
50. $\sqrt[3]{\frac{1}{2}}=\sqrt[3]{\frac{4}{2}}$
51. $\sqrt{x^3}=x\sqrt{x}$
52. $2\sqrt{x}$
53. $\sqrt{2x}$
54. $5^{\frac{1}{2}}$
55. $2^{\frac{3}{2}}$
56. $a^{\frac{1}{3}}$
57. $a^{-\frac{1}{2}}$
58. 36
59. $\frac{4}{9}$
60. 4
61. 2
62. $\frac{1}{2}$
63. $\frac{3|x|^3}{|y|}$
64. $\frac{b^8}{4a^4}$
65. –9
66. $5\sqrt{2}$
67. $6\sqrt{3}$
68. $3\sqrt[3]{2}$
69. $2\sqrt{13}$
70. 0
71. $x^2\sqrt{x}$
72. $2|x|^3$
73. $4x\sqrt{2x}$
74. a if $a\ge 0$, $-a$ if $a<0$
75. $a\sqrt{a}$
76. $a\sqrt[3]{a^2}$
77. $4\sqrt{2}$
78. $4\sqrt{3}$
79. $3\sqrt{x^2-y^2}$
80. $3\sqrt{x^2+y^2}$
81. $3|x+y|$
82. $4(x+y)$
83. $\frac{\sqrt{6}}{3}$
84. $\frac{\sqrt{5}}{5}$
85. $\sqrt{3}$

86. $\sqrt{ab}/b$
87. $\sqrt[3]{18}/3$
88. $3\sqrt{2}/2$
89. $3\sqrt{2}-3$
90. $\frac{\sqrt{3}+3}{-2}$
91. $-7-4\sqrt{3}$

TOPIC 4: LINEAR EQUATIONS and INEQUALITIES

A. Solving one linear equation in one variable:
Add or subtract the same value on each side of the equation, or multiply or divide each side by the same value, with the goal of placing the variable alone on one side. If there are one or more fractions, it may be desirable to eliminate them by multiplying both sides by the common denominator. If the equation is a proportion, you may wish to cross-multiply.

Problems 1-15: Solve:

1. $2x = 9$
2. $3 = \frac{6x}{5}$
3. $3x + 7 = 6$
4. $\frac{x}{3} = \frac{5}{4}$
5. $5 - x = 9$
6. $x = \frac{2x}{5} + 1$
7. $4x - 6 = x$
8. $\frac{x-1}{x+1} = \frac{6}{7}$
9. $x - 4 = \frac{x}{2} + 1$
10. $\frac{3x}{2x+1} = \frac{5}{2}$
11. $6 - 4x = x$
12. $\frac{3x-2}{2x+1} = 4$
13. $\frac{x+3}{2x-1} = 2$
14. $7x - 5 = 2x + 10$
15. $\frac{1}{3} = \frac{x}{x+8}$

To solve a linear equation for one variable in terms of the other, do the same as above:

example: Solve for F: $C = \frac{5}{9}(F - 32)$
Multiply by $\frac{9}{5}$: $\frac{9}{5}C = F - 32$
Add 32: $\frac{9}{5}C + 32 = F$
Thus, $F = \frac{9}{5}C + 32$
example: Solve for b: $a + b = 90$
Subtract a: $b = 90 - a$

Problems 16-21: Solve for the indicated variable in terms of the other(s):

16. $a + b = 180$; $b =$
17. $2a + 2b = 180$; $b =$
18. $P = 2b + 2h$; $b =$
19. $y = 3x - 2$; $x =$
20. $y = 4 - x$; $x =$
21. $y = \frac{2}{3}x + 1$; $x =$

B. Solving a pair of linear equations in two variables:
The solution consists of an ordered pair, an infinite number of ordered pairs, or no solution.

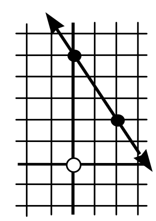
Problems 22-28: Solve for the common solution(s) by substitution or linear combinations:

22. $x + 2y = 7$, $3x - y = 28$
23. $x + y = 5$, $x - y = -3$
24. $2x - y = -9$, $x = 8$
25. $2x - y = 1$, $y = x - 5$
26. $2x - 3y = 5$, $3x + 5y = 1$
27. $4x - 1 = y$, $4x + y = 1$
28. $x + y = 3$, $x + y = 1$
29. $2x - y = 3$, $6x - 9 = 3y$

C. Analytic geometry of one linear equation in two variables:

The graph of $y = mx + b$ is a line with slope m and y-intercept b. To draw the graph, find one point on it (such as (0, b)) and then use the slope to find another point. Draw the line joining the two points.

example: $y = \frac{-3}{2}x + 5$ has slope $-3/2$ and y-intercept 5.
To graph the line, locate (0, 5). From that point, go down 3 (top of slope fraction), and over (right) 2 (bottom of fraction) to find a second point.
Draw the line joining the points.

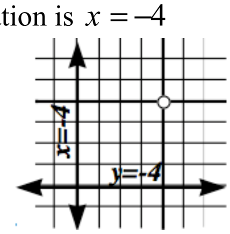


Problems 30-34: Find slope and y-intercept, and sketch the graph:

30. $y = x + 4$
31. $y = -\frac{1}{2}x - 3$
32. $2y = 4x - 8$
33. $x - y = -1$
34. $x = -3y + 2$

A vertical line has no slope, and its equation can be written so it looks like $x = k$ (where k is a number). A horizontal line <u>has zero slope</u>, and its equation looks like $y = k$.

example: Graph on the same graph: $x + 3 = -1$ and $1 + y = -3$.
The first equation is $x = -4$



The second is $y = -4$

Problems 35-36: Graph and write an equation for…

35. The line through (-1, 4) and (-1, 2)
36. The horizontal line through (4, -1)

D. Analytic geometry of two linear equations in two variables:

Two distinct lines in a plane are either parallel or intersecting. They are parallel if and only if they have the same slope, and hence the equations of the lines have no common solutions. If the lines have unequal slopes, they intersect in one point and their equations have exactly one common solution. (They are perpendicular if and only if their slopes are negative reciprocals, or one is horizontal and the other is vertical.) If one equation is a multiple of the other, each equation has the same graph, and every solution of one equation is a solution of the other.

Problems 37-44: For each pair of equations in problems 22 to 29, tell whether the lines are parallel, perpendicular, intersecting but not perpendicular, or the same line:

37. Problem 22	41. Problem 26
38. Problem 23	42. Problem 27
39. Problem 24	43. Problem 28
40. Problem 25	44. Problem 29

E. Solution of a one-variable equation reducible to a linear equation:

Some equations which do not appear to be linear can be solved by using a related linear equation:

example: $|3-x|=2$

Since the absolute value of both 2 and –2 is 2, 3-*x* can be either 2 or –2. Write these two equations and solve each:

$3-x=2$ or $3-x=-2$
$-x=-1$ $\quad$ $-x=-5$
$x=1$ $\quad$ $x=5$

Problems 45-49: Solve:

45. $\lvert x\rvert=3$	48. $\lvert 2-3x\rvert=0$
46. $\lvert x\rvert=-1$	49. $\lvert x+2\rvert=1$
47. $\lvert x-1\rvert=3$	

example: $\sqrt{2x-1}=5$

Square both sides: $2x-1=25$

Solve: $2x=26$
$x=13$

Be sure to check answer(s):

$\sqrt{2x-1}=\sqrt{2\bullet 13-1}$
$=\sqrt{25}=5$ (Check)

example: $\sqrt{x}=-3$

Square: $x=9$

Check: $\sqrt{x}=\sqrt{9}=3\neq-3$

There is no solution, since 9 doesn't satisfy the original equation (it is false that $\sqrt{9}=-3$).

Problems 50-52: Solve and check:

50. $\sqrt{3-x}=4$	52. $3=\sqrt{3x-2}$
51. $\sqrt{2x+1}=\sqrt{x-3}$	

F. Linear inequalities:

Rules for inequalities:

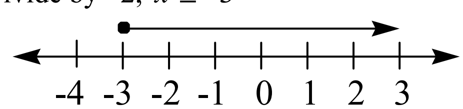
if a > b, then:	if a < b, then:
$a+c>b+c$	$a+c<b+c$
$a-c>b-c$	$a-c<b-c$
$ac>bc$ (*if* $c>0$)	$ac<bc$ (*if* $c>0$)
$ac<bc$ (*if* $c<0$)	$ac>bc$ (*if* $c<0$)
$\frac{a}{c}>\frac{b}{c}$ (*if* $c>0$)	$\frac{a}{c}<\frac{b}{c}$ (*if* $c>0$)
$\frac{a}{c}<\frac{b}{c}$ (*if* $c<0$)	$\frac{a}{c}>\frac{b}{c}$ (*if* $c<0$)

example: One variable graph: solve and graph on a number line: $1-2x\leq 7$ (This is an abbreviation for $\{x: 1-2x\leq 7\}$)

Subtract 1, get $-2x\leq 6$

Divide by –2, $x\geq -3$

Graph:



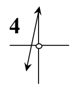
Problems 53-59: Solve and graph on a number line:

53. $x-3>4$	57. $4-2x<6$
54. $4x<2$	58. $5-x>x-3$
55. $2x+1\leq 6$	59. $x>1+4$
56. $3<x-3$	

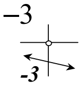
Answers:

1. $9/2$	9. 10	17. $90-a$
2. $5/2$	10. $-5/4$	18. $p/2-h=\frac{p-2h}{2}$
3. $-1/3$	11. $6/5$	19. $(y+2)/3$
4. $15/4$	12. $-6/5$	20. $4-y$
5. – 4	13. $5/3$	21. $3(y-1)/2$
6. $5/3$	14. 3	22. (9, -1)
7. 2	15. 4	23. (1, 4)
8. 13	16. $180-a$	

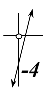
24. (8, 25)
25. (-4, -9)
26. $\left(\frac{28}{19}, -\frac{13}{19}\right)$
27. $\left(\frac{1}{4}, 0\right)$
28. no solution
29. $(a,\ 2a-3)$, where a is any number; infinite number of solutions
30. $m=1,\ b=4$



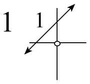
31. $m=-\frac{1}{2},\ b=-3$



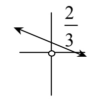
32. $m=2,\ b=-4$



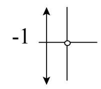
33. $m=1,\ b=1$



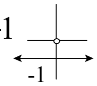
34. $m=-\frac{1}{3},\ b=\frac{2}{3}$



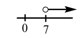
35. $x=-1$



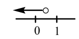
36. $y=-1$



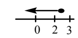
37. intersecting, not⊥
38. ⊥
39. intersecting, not⊥
40. intersecting, not⊥
41. intersecting, not⊥
42. intersecting, not⊥
43. parallel
44. same line
45. –3, 3
46. no solution
47. –2, 4
48. $\frac{2}{3}$
49. –3, -1
50. –13
51. no solution in real numbers
52. $\frac{11}{3}$
53. $x > 7$



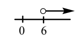
54. $x < \frac{1}{2}$



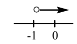
55. $x \le \frac{5}{2}$



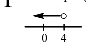
56. $x > 6$



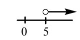
57. $x > -1$



58. $x < 4$



59. $x > 5$



TOPIC 5: QUADRATIC POLYNOMIALS, EQUATIONS, and INEQUALITIES

A. <u>Multiplying polynomials</u>:

example: $(x+2)(x+3)=x^2+5x+6$
example: $(2x-1)(x+2)=2x^2+3x-2$
example: $(x-5)(x+5)=x^2-25$
example: $-4(x-3)=-4x+12$
example: $(x+2)(x^2-2x+4)=x^3+8$
example: $(3x-4)^2=9x^2-24x+16$
example: $(x+3)(a-5)=ax-5x+3a-15$

Problems 1-10: Multiply:

1. $(x+3)^2=$
2. $(x-3)^2=$
3. $(x+3)(x-3)=$
4. $(2x+3)(2x-3)=$
5. $(x-4)(x-2)=$
6. $-6x(3-x)=$
7. $(2x-1)(4x^2+2x+1)=$
8. $\left(x-\frac{1}{2}\right)^2=$
9. $(x-1)(x+3)=$
10. $(x^2-1)(x^2+3)=$

B. <u>Factoring</u>:

<u>Monomial factors</u>: $ab+ac=a(b+c)$

example: $x^2-x=x(x-1)$
example: $4x^2y+6xy=2xy(2x+3)$

<u>Difference of two squares</u>: $a^2-b^2=(a+b)(a-b)$

example: $9x^2-4=(3x+2)(3x-2)$

<u>Trinomial square</u>: $a^2+2ab+b^2=(a+b)^2$
$a^2-2ab+b^2=(a-b)^2$

example: $x^2-6x+9=(x-3)^2$

<u>Trinomial</u>:

example: $x^2-x-2=(x-2)(x+1)$
example: $6x^2-7x-3=(3x+1)(2x-3)$

<u>Sum and difference of two cubes</u>:
$a^3+b^3=(a+b)(a^2-ab+b^2)$
$a^3-b^3=(a-b)(a^2+ab+b^2)$

example: $x^3-64=x^3-4^3=(x-4)(x^2+4x+16)$

Problems 11-27: Factor:

11. $a^2+ab=$
12. $a^3-a^2b+ab^2=$
13. $8x^2-2=$
14. $x^2-10x+25=$
15. $-4xy+10x^2=$
16. $2x^2-3x-5=$
17. $x^2-x-6=$
18. $x^2y-y^2x=$
19. $8x^3+1=$
20. $x^2-3x-10=$
21. $2x^2-x=$
22. $8x^3+8x^2+2x=$
23. $9x^2+12x+4=$
24. $6x^3y^2-9x^4y=$
25. $1-x-2x^2=$
26. $3x^2-10x+3=$
27. $x^4+3x^2-4=$

C. <u>Solving quadratic equations by factoring</u>:
If $ab=0$, then $a=0$ or $b=0$.

example: if $(3-x)(x+2)=0$ then $(3-x)=0$ or $(x+2)=0$ and thus, $x=3$ or $x=-2$

Note: there must be a zero on one side of the equation to solve by the factoring method.

example: $6x^2 = 3x$

Rewrite: $6x^2 - 3x = 0$

Factor: $3x(2x-1) = 0$

So $3x = 0$ or $(2x-1) = 0$

Thus $x = 0$ or $x = \frac{1}{2}$

Problems 28-39: Solve by factoring:

28. $x(x-3)=0$	34. $(x+2)(x-3)=0$
29. $x^2-2x=0$	35. $(2x+1)(3x-2)=0$
30. $2x^2=x$	36. $6x^2=x+2$
31. $3x(x+4)=0$	37. $9+x^2=6x$
32. $x^2=2-x$	38. $1-x=2x^2$
33. $x^2+x=6$	39. $x^2-x-6=0$

D. Completing the square:

x^2+bx will be the square of a binomial when c is added, if c is found as follows: find half the x coefficient and square it–this is c.

Thus, $c = \left(\frac{b}{2}\right)^2 = \frac{b^2}{4}$, and

$$x^2+bx+c = x^2+bx+\frac{b^2}{4} = \left(x+\frac{b}{2}\right)^2$$

example: x^2+5x

Half of 5 is $\frac{5}{2}$, and $\left(\frac{5}{2}\right)^2 = \frac{25}{4}$, which must be added to complete the square:

$x^2+5x+\frac{25}{4} = \left(x+\frac{5}{2}\right)^2$

If the coefficient of x^2 is not 1, factor it so that it is.

example: $3x^2 - x = 3(x^2 - \frac{1}{3}x)$

Half of $-\frac{1}{3}$ is $-\frac{1}{6}$, and $\left(-\frac{1}{6}\right)^2 = \frac{1}{36}$, so

$\left(x^2-\frac{1}{3}x+\frac{1}{36}\right) = \left(x-\frac{1}{6}\right)^2$, and

$3\left(x^2-\frac{1}{3}x+\frac{1}{36}\right) = 3x^2-x+\frac{3}{36}$

Thus $\frac{3}{36}$ $\left(\text{or } \frac{1}{12}\right)$ must be added to $3x^2-x$ to complete the square.

Problems 40-43: Complete the square, and tell what must be added:

40. x^2-10x	42. $x^2-\frac{3}{2}x$
41. x^2+x	43. $2x^2+8x$

E. The quadratic formula:

If a quadratic equation looks like $ax^2+bx+c=0$, then the roots (solutions) can be found by using the quadratic formula: $x = \frac{-b\pm\sqrt{b^2-4ac}}{2a}$

Problems 44-49: Solve:

44. $x^2-x-6=0$	47. $x^2-3x-4=0$
45. $x^2+2x=-1$	48. $x^2+x-5=0$
46. $2x^2-x-2=0$	49. $x^2+x=1$

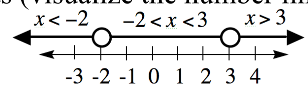
F. Quadratic inequalities:

example: Solve $x^2-x<6$. First make one side zero: $x^2-x-6<0$

Factor: $(x-3)(x+2)<0$.

If $(x-3)=0$ or $(x+2)=0$ then $x=3$ or $x=-2$.

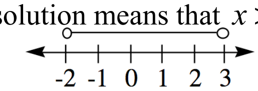
These two numbers (3 and –2) split the real numbers into three sets (visualize the number line):



x	$(x-3)$	$(x+2)$	$(x-3)\times(x+2)$	Solution ?
$x<-2$	neg	neg	pos	no
$-2<x<3$	neg	pos	neg	yes
$x>3$	pos	pos	pos	no

Therefore, if $(x-3)(x+2)<0$, then $-2<x<3$

Note that this solution means that $x>-2$ and $x<3$



Problems 50-54: Solve, and graph on a number line:

50. $x^2-x-6>0$	53. $x>x^2$
51. $x^2+2x<0$	54. $2x^2+x-1>0$
52. $x^2-2x<-1$	

G. Complex numbers:

$\sqrt{-1}$ is defined to be i, so $i^2=-1$

example: $i^3 = i^2 \bullet i = -1 \bullet i = -i$

55. Find the value of i^4.

A complex number is of the form $a+bi$, where a and b are real numbers. a is called the real part and b is the imaginary part. If b is zero, $a+bi$ is a real number. If $a=0$, then $a+bi$ is pure imaginary.

Complex number operations:

example: $(3+i)+(2-3i) = 5-2i$

example: $(3+i)-(2-3i) = 1+4i$

example: $(3+i)(2-3i) = 6-7i-3i^2$
$= 6-7i+3 = 9-7i$

example: $\frac{3+i}{2-3i} = \frac{3+i}{2-3i} \bullet \frac{2+3i}{2+3i} = \frac{6+11i-3}{4+9} = \frac{3+11i}{13} = \frac{3}{13}+\frac{11}{13}i$

Problems 56-65: Write each of the following so the answer is $a+bi$:

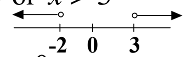
56. $(3+2i)(3-2i)=$	61. $i^6=$
57. $(3+2i)+(3-2i)=$	62. $i^7=$
58. $(3+2i)-(3-2i)=$	63. $i^8=$
59. $(3+2i)\div(3-2i)=$	64. $i^{1991}=$
60. $i^5=$	65. $\frac{1}{i}=$

Problems 66-67: Solve and write the answer as $a+bi$:

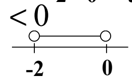
66. $x^2+2x+5=0$	67. $x^2+x+2=0$

Answers:

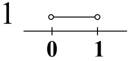
1. x^2+6x+9
2. x^2-6x+9
3. x^2-9
4. $4x^2-9$
5. x^2-6x+8
6. $-18x+6x^2$
7. $8x^3-1$
8. $x^2-x+\frac{1}{4}$
9. x^2+2x-3
10. x^4+2x^2-3
11. $a(a+b)$
12. $a(a^2-ab+b^2)$
13. $2(2x+1)(2x-1)$
14. $(x-5)^2$
15. $2x(-2y+5x)$
16. $(2x-5)(x+1)$
17. $(x-3)(x+2)$
18. $xy(x-y)$
19. $(2x+1)(4x^2-2x+1)$
20. $(x-5)(x+2)$
21. $x(2x-1)$
22. $2x(2x+1)^2$
23. $(3x+2)^2$
24. $3x^3y(2y-3x)$
25. $(1-2x)(1+x)$
26. $(3x-1)(x-3)$
27. $(x^2+4)(x+1)(x-1)$
28. 0, 3
29. 0, 2
30. 0, $\frac{1}{2}$
31. –4, 0
32. –2, 1
33. –3, 2
34. –2, 3
35. $-\frac{1}{2}$, $\frac{2}{3}$
36. $-\frac{1}{2}$, $\frac{2}{3}$
37. 3
38. –1, $\frac{1}{2}$
39. –2, 3
40. $(x-5)^2$, add 25
41. $(x+\frac{1}{2})^2$, add $\frac{1}{4}$
42. $(x-\frac{3}{4})^2$, add $\frac{9}{16}$
43. $2(x+2)^2$, add 8
44. –2, 3
45. –1
46. $\frac{1\pm\sqrt{17}}{4}$
47. –1, 4
48. $\frac{-1\pm\sqrt{21}}{2}$
49. $\frac{-1\pm\sqrt{5}}{2}$
50. $x<-2$ or $x>3$



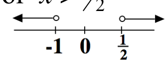
51. $-2<x<0$



52. no solution, no graph
53. $0<x<1$



54. $x<-1$ or $x>\frac{1}{2}$

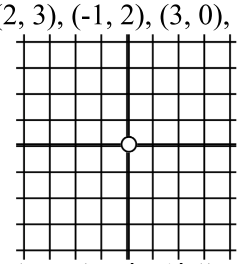


55. 1
56. 13
57. 6
58. $4i$
59. $\frac{5}{13}+\frac{12}{13}i$
60. i
61. –1
62. $-i$
63. 1
64. $-i$
65. $-i$
66. $-1\pm2i$
67. $-\frac{1}{2}\pm\frac{\sqrt{7}}{2}i$

TOPIC 6: GRAPHING and the COORDINATE PLANE

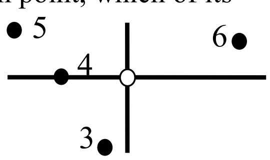
A. Graphing points:

1. Join the following points in the given order: (-3, -2), (1, -4), (3, 0), (2, 3), (-1, 2), (3, 0), (-3, -2), (-1, 2), (1, -4)



2. In what quadrant does the point (a, b) lie, if $a>0$ and $b<0$?

Problems 3-6: For each given point, which of its coordinates, *x* or *y*, is larger?



B. Distance between points:

The distance between the points $P_1(x_1,\ y_1)$ and $P_2(x_2,\ y_2)$ is found by using the Pythagorean Theorem, which gives

$$P_1P_2=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}\ .$$

> *example:* $A(3, -1)$, $B(-2, 4)$
>
> $$AB=\sqrt{(4-(-1))^2+(-2-3)^2}=$$
> $$\sqrt{5^2+(-5)^2}=\sqrt{50}=\sqrt{25}\sqrt{2}=5\sqrt{2}$$

Problems 7-10: Find the length of the segment joining the given points:

7. (4, 0), (0, -3)
8. (-1, 2), (-1, 5)
9. (2, -4), (0, 1)
10. $(-\sqrt{3},\ -5)$, $(3\sqrt{3},\ -6)$

C. Linear equations in two variables, slope, intercepts, and graphing:

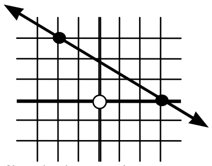
The line joining the points $P_1(x_1,\ y_1)$ and $P_2(x_2,\ y_2)$ has slope $\frac{y_2-y_1}{x_2-x_1}$.

> *example:* A(3,-1), B(-2,4) slope of
>
> $$\overline{AB}=\frac{4-(-1)}{-2-3}=\frac{5}{-5}=-1$$

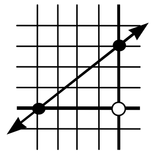
Problems 11-15: Find the slope of the line joining the given points:

11. (-3, 1), and (-1, -4)
12. (0, 2), and (-3, 5)
13. (3, -1), and (5, -1)

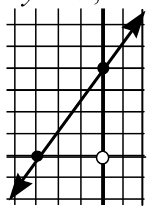
14.



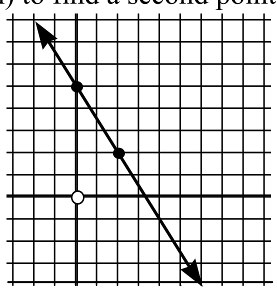
15.



To find the x-intercept (x-axis crossing) of an equation, let y be zero and solve for x. For the y-intercept, let x be zero and solve for y.

> *example:* $3y-4x=12$ if $x=0$, $y=4$ so y-intercept is 4. If $y=0$, $x=-3$ so x-intercept is –3.
>
> 

The graph of $y=mx+b$ is a line with slope m and y-intercept b. To draw the graph, find one point on it (such as (0, b)) and then use the slope to find another point. Draw the line joining the two.

> *example:* $y=\frac{-3}{2}x+5$ has slope $-3/2$ and y-intercept 5. To graph the line, locate (0, 5). From that point, go down 3 (top of slope fraction), and over (right) 2 (bottom of fraction) to find a second point. Join.
>
> 

Problems 16-20: Find the slope and y-intercept, and sketch the graph:

16. $y=x+4$
17. $y=-\frac{1}{2}x-3$
18. $2y=4x-8$
19. $x-y=-1$
20. $x=-3y+2$

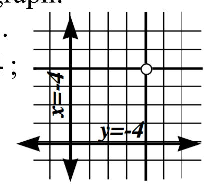
To find an equation of a non-vertical line, it is necessary to know its slope and one of its points. Write the slope of the line through (x,y) and the known point, then write an equation which says that this slope equals the known slope.

> *example:* Find an equation of the line through (-4, 1) and (-2, 0).
>
> Slope = $\frac{1-0}{-4+2}=\frac{1}{-2}$
>
> Using (-2, 0) and (x,y),
>
> Slope = $\frac{y-0}{x+2}=\frac{1}{-2}$; cross multiply, get
>
> $-2y=x+2$, or $y=-\frac{1}{2}x-1$

Problems 21-25: Find an equation of the line:

21. Through (-3, 1) and (-1, -4)
22. Through (0, -2) and (-3, -5)
23. Through (3, -1) and (5, -1)
24. Through (8, 0), with slope –1
25. Through (0, -5), with slope $2/3$

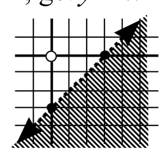
A vertical line has no slope, and its equation can be written so it looks like $x=k$ (where k is a number). A horizontal line has zero slope, and its equation looks like $y=k$.

> *example*: Graph on the same graph:
>
> $x+3=-1$ and $1+y=-3$.
>
> The first equation is $x=-4$; the second is $y=-4$.
>
> 
>

Problems 26-27: Graph and write equation for:

26. The line through (-1, 4) and (-1, 2)
27. Horizontal line through (4, -1)

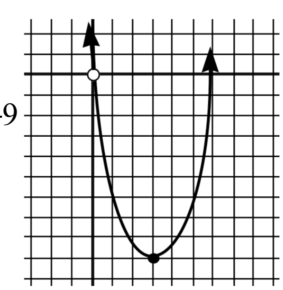
D. <u>Linear inequalities in two variables</u>:

> *example:* Two variable graph: graph solution on a number plane: $x-y>3$
>
> (This is an abbreviation for $\{(x,y):\ x-y>3\}$.
>
> Subtract x, multiply by –1, get $y<x-3$.
>
> Graph $y=x-3$, but draw a dotted line, and shade the side where $y<x-3$:
>
> 

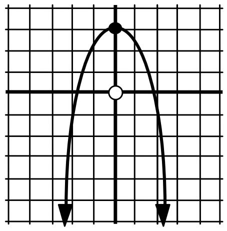
28. $y<3$
29. $y>x$
30. $y\geq\frac{2}{3}x+2$
31. $x<y+1$
32. $x+y<3$
33. $2x-y>1$

E. <u>Graphing quadratic equations</u>:

The graph of $y=ax^2+bx+c$ is a parabola, opening upward (if $a>0$) or downward (if $a<0$), and with line of symmetry. $x=\frac{-b}{2a}$, also called axis of symmetry. To find the vertex V(h, k) of the parabola, $h=\frac{-b}{2a}$ (since V is on the axis of symmetry), and k is the value of y when h is substituted for x.

> *example:* $y=x^2-6x$
>
> $a=1,\ b=-6,\ c=0$
>
> $x=\frac{-b}{2a}=\frac{6}{2}=3$
>
> Axis: $h=3$,
>
> $k=3^2-18=-9$
>
> Thus, vertex is $(3,-9)$
>
> 

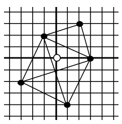
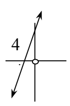
example: $y = 3 - x^2$ V(0, 3),
Axis: $x = 0$, y-intercept: if $x = 0$, $y = 3 - 0^2 = 3$
x-intercept: if $y = 0$, $0 = 3 - x^2$,
so $3 = x^2$, and $x = \pm\sqrt{3}$

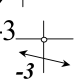


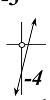
Problems 34-40: Sketch the graph:

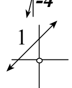
34. $y = x^2$
35. $y = -x^2$
36. $y = x^2 + 1$
37. $y = x^2 - 3$
38. $y = (x+1)^2$
39. $y = (x-2)^2 - 1$
40. $y = (x+2)(x-1)$

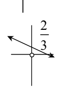
Answers:

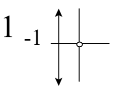
1. 
2. IV
3. x
4. y
5. y
6. x
7. 5
8. 3
9. $\sqrt{29}$
10. 7
11. $-5/2$
12. -1
13. 0
14. $-3/5$
15. $3/4$
16. $m = 1,\ b = 4$ 

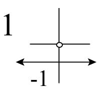
17. $m = -1/2,\ b = -3$ 

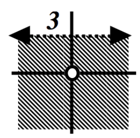
18. $m = 2,\ b = -4$ 

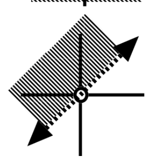
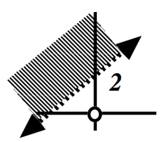
19. $m = 1,\ b = 1$ 

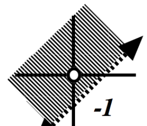
20. $m = -1/3,\ b = 2/3$ 

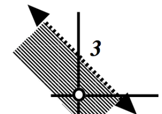
21. $y = -5/2\,x - 13/2$
22. $y = x - 2$
23. $y = -1$
24. $y = -x + 8$
25. $y = 2/3\,x - 5$
26. $x = -1$ 

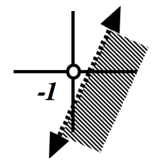
27. $y = -1$ 

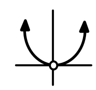
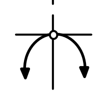
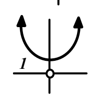
28. 

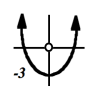
29. 
30. 

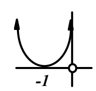
31. 

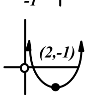
32. 

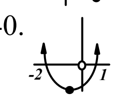
33. 

34. 
35. 
36. 

37. 

38. 

39. 

40. 

TOPIC 7: LOGARITHMS and FUNCTIONS

A. Functions:
The area A of a square depends on its side length s, and we say A is a function of s, and write ‘A= $f(s)$’; for short, we read this as ‘A= f of s.’ There are many functions of s. The one here is s^2. We write this $f(s) = s^2$ and can translate: ‘the function of s we’re talking about is s^2’. Sometimes we write $A(s) = s^2$. This says the area A is a function of s, and specifically, it is s^2.

B. Function values and substitution:
If $A(s) = s^2$, A(3), read ‘A of 3’, means replace every s in $A(s) = s^2$ with 3, and find A when s is 3. When we do this, we find $A(3) = 3^2 = 9$.

example: $g(x)$ is given: $y = g(x) = \pi x^2$

example: $g(3) = \pi \bullet 3^2 = 9\pi$
example: $g(7) = \pi \bullet 7^2 = 49\pi$
example: $g(a) = \pi a^2$
example: $g(x+h) = \pi(x+h)^2 = \pi x^2 + 2\pi xh + \pi h^2$

1. Given $y = f(x) = 3x - 2$.
   Complete these ordered pairs: (3,___), (0,___), ($\frac{1}{2}$,___), (___, 10), (___,-1) ($x-1$,___)

Problems 2-10: Given $f(x) = x^2 - 4x + 2$. Find:

2. $f(0) =$
3. $f(1) =$
4. $f(-1) =$
5. $f(-x) =$
6. $-f(x) =$
7. $f(x) - 2 =$
8. $f(x-2) =$
9. $2f(x) =$
10. $f(2x) =$

Problems 11-15: Given $f(x) = \frac{x}{x+1}$. Find:

11. $f(1) =$
12. $f(-2) =$
13. $f(0) =$
14. $f(-1) =$
15. $f(x-1) =$

example: If $k(x) = x^2 - 4x$, for what x is $k(x) = 0$?
If $k(x) = 0$, then $x^2 - 4x = 0$ and since $x^2 - 4x = x(x-4) = 0$, x can be either 0 or 4.
(These values of x: 0 and 4, are called 'zeros of the function', because each makes the function zero.)

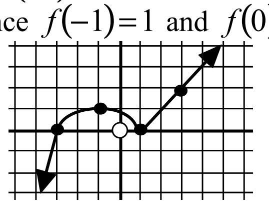
Problems 16-19: Find all real zeros of:

16. $x(x+1)$
17. $2x^2 - x - 3$
18. $x^2 - 16x + 64$
19. $x^2 + x + 2$

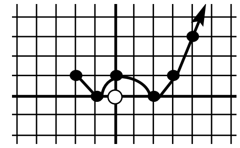
Problems 20-23: Given $f(x) = x^2 - 4x + 2$, find real x so that:

20. $f(x) = -2$
21. $f(x) = 2$
22. $f(x) = -3$
23. x is a zero of $f(x)$

Since $y = f(x)$, the values of y are the values of the function which correspond to specific values of x. The heights of the graph above (or below) the x-axis are the values of y and so also of the function. Thus for this graph, $f(3)$ is the height (value) of the function at $x = 3$ and the value is 2: At $x = -3$, the value (height) of $f(x)$ is zero; in other words, $f(-3) = 0$. Note that $f(3) > f(-3)$, since $2 > 0$, and that $f(0) < f(-1)$, since $f(-1) = 1$ and $f(0) < 1$.



Problems 24-28: For this graph, tell whether the statement is true or false:



24. $g(-1) = g(0)$
25. $g(0) = g(3)$
26. $g(1) > g(-1)$
27. $g(-2) > g(1)$
28. $g(2) < g(0) < g(4)$

C. Logarithms and exponents:

Exponential form: $2^3 = 8$
Logarithmic form: $\log_2 8 = 3$

Both of the equations above say the same thing. '$\log_2 8 = 3$' is read 'log base two of eight equals three' and translates 'the power of 2 which gives 8 is 3'.

Problems 29-32: Write the following information in both exponential and logarithmic forms:

29. The power of 3 which gives 9 is 2.
30. The power of x which gives x^3 is 3
31. 10 to the power –2 is $\frac{1}{100}$.
32. $\frac{1}{2}$ is the power of 169 which gives 13.

Problems 33-38: Write in logarithmic form:

33. $4^3 = 64$
34. $3^0 = 1$
35. $25 = 5^2$
36. $\frac{1}{10} = 10^{-1}$
37. $a^b = c$
38. $y = 3^x$

Problems 39-44: Write in exponential form:

39. $\log_3 9 = 2$
40. $\log_3 1 = 0$
41. $5 = \log_2 32$
42. $1 = \log_4 4$
43. $y = \log_a x$
44. $\log_b a = 2$

Problems 45-50: Find the value:

45. $2^{10} =$
46. $\log_4 4^{10} =$
47. $\log_6 6 =$
48. $\frac{6^{10}}{3^{10}} =$
49. $\log_{49} 7 =$
50. $\log_7 49 =$

D. Logarithm and exponent rules:

exponent rules: all quantities real)	log rules: (base any positive real number except 1)
$a^b \bullet a^c = a^{b+c}$	$\log ab = \log a + \log b$
$\frac{a^b}{a^c} = a^{b-c}$	$\log \frac{a}{b} = \log a - \log b$
$(a^b)^c = a^{bc}$	$\log a^b = b \log a$
$(ab)^c = a^c b^c$	$\log_a a^b = b$
$\left(\frac{a}{b}\right)^c = \frac{a^c}{b^c}$	$a^{(\log_a b)} = b$
$a^0 = 1$ (if $a \neq 0$)	$\log_a b = \frac{\log_c b}{\log_c a}$
$a^{-b} = \frac{1}{a^b}$	
$a^{\frac{p}{r}} = \sqrt[r]{a^p} = (\sqrt[r]{a})^p$	
(think of $\frac{p}{r}$ as $\frac{\text{power}}{\text{root}}$)	(base change rule)

Problems 51-52: Given $\log_2 1024 = 10$, find:

51. $\log_2 1024^5 =$
52. $\log_2 \sqrt{1024} =$

Problems 53-63: Solve for *x* in terms of *y* and *z*:

53. $3^x = 3^y \bullet 3^z$
54. $9^y = \frac{3^z}{3^x}$
55. $x^3 = y$
56. $3^x = y$
57. $\log x^2 = 3\log y$
58. $\log x = 2\log y - \log z$
59. $3\log x = \log y$
60. $\log x = \log y + \log z$
61. $\log\sqrt{x} + \log\sqrt[3]{y} = \log z^2$
62. $\log_7 3 = y;\ \log_7 2 = z;\ x = \log_3 2$
63. $y = \log_a 9;\ x = \log_a 3$

E. <u>Logarithmic and exponential equations</u>:

Problems 64-93: Use the exponent and log rules to find the value of *x*:

64. $6^{2x} = 6^3$
65. $2^{2x} = 2^3$
66. $4^x = 8$
67. $9^x = 27^{x-1}$
68. $\log_3 x = \log_3 6$
69. $\log_3 4x = \log_3 6$
70. $4^3 \bullet 4^5 = 4^x$
71. $3^{-2} = x$
72. $\frac{3^x}{3} = 3^0$
73. $\left(5^2\right)^3 = 5^x$
74. $5^{x+1} = 1$
75. $\log_3 3^7 = x$
76. $6^{(\log_6 x)} = 8$
77. $\log_{10} x = \log_{10} 4 + \log_{10} 2$
78. $\log_3 2x = \log_3 8 + \log_3 4 - 4\log_3 2$
79. $\log_x 25 = 2$
80. $3\log_a 4 = \log_a x$
81. $\log(2x-6) = \log(6-x)$
82. $\frac{\log_x 3}{\log_x 4} = \log_4 3$
83. $\log_3\left(27 \bullet 3^{-4}\right) = x$
84. $\frac{\log_3 81}{\log_3 27} - \log_3 \frac{81}{27} = x$
85. $\log_4 \sqrt[3]{30} = x \log_4 30$
86. $\log_2 \frac{1}{32} = x$
87. $\log_{16} x = \frac{3}{2}$
88. $\log_4 64 = x$
89. $\sqrt[4]{5} = 5^x$
90. $27^x = \left(\frac{1}{9}\right)^3$
91. $4^{10} = 2^x$
92. $2^x = 3$
93. $3 \bullet 2^x = 4$

<u>Answers</u>:

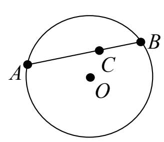
1. 7, -2, $-\frac{1}{2}$, 4, $\frac{1}{3}$, $3x-5$
2. 2
3. -1
4. 7
5. $x^2 + 4x + 2$
6. $-x^2 + 4x - 2$
7. $x^2 - 4x$
8. $x^2 - 8x + 14$
9. $2x^2 - 8x + 4$
10. $4x^2 - 8x + 2$
11. $\frac{1}{2}$
12. 2
13. 0
14. no value
15. $\frac{x-1}{x}$
16. -1, 0
17. -1, $\frac{3}{2}$
18. 8
19. none
20. 2
21. 0, 4
22. none
23. $2 \pm \sqrt{2}$
24. F
25. T
26. T
27. T
28. T
29. $3^2 = 9,\ \log_3 9 = 2$
30. $x^3 = x^3,\ \log_x x^3 = 3$
31. $10^{-2} = \frac{1}{100},\ \log_{10} \frac{1}{100} = -2$
32. $169^{1/2} = 13,\ \log_{169} 13 = \frac{1}{2}$
33. $\log_4 64 = 3$
34. $\log_3 1 = 0$
35. $\log_5 25 = 2$
36. $\log_{10} \frac{1}{10} = -1$
37. $\log_a c = b$
38. $\log_3 y = x$
39. $3^2 = 9$
40. $3^0 = 1$
41. $2^5 = 32$
42. $4^1 = 4$
43. $a^y = x$
44. $b^2 = a$
45. 1024
46. 10
47. 1
48. $2^{10} = 1024$
49. $\frac{1}{2}$
50. 2
51. 50
52. 5
53. $y + z$
54. $z - 2y$
55. $y^{1/3} = \sqrt[3]{y}$
56. $\log_3 y$
57. $y^{3/2} = y\sqrt{y}$
58. $\frac{y^2}{z}$
59. $\sqrt[3]{y}$
60. yz
61. $\frac{z^4}{\sqrt[3]{y^2}}$
62. $\frac{z}{y}$
63. $\frac{y}{2}$
64. $\frac{3}{2}$
65. $\frac{3}{2}$
66. $\frac{3}{2}$
67. 3
68. 6
69. $\frac{3}{2}$
70. 8
71. $\frac{1}{9}$
72. 1
73. 6
74. -1
75. 7
76. 8
77. 8
78. 1
79. 5

80. 64
81. 4
82. any real number > 0 and $\neq 1$
83. –1
84. $\frac{1}{3}$
85. $\frac{1}{3}$
86. –5
87. 64
88. 3
89. $\frac{1}{4}$
90. –2
91. 20
92. $\frac{\log 3}{\log 2}$
93. $\frac{\log 4 - \log 3}{\log 2}$ (any base; if base=2, $x = 2 - \log_2 3$)

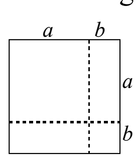
TOPIC 8: WORD PROBLEMS

1. $\frac{2}{3}$ of $\frac{1}{6}$ of $\frac{3}{4}$ of a number is 12. What is the number?
2. On the number line, points *P* and *Q* and 2 units apart. *Q* has coordinate *x*. What are the possible coordinates of *P*?
3. What is the number, which when multiplied by 32, gives $32 \bullet 46$?
4. If you square a certain number, you get 9^2. What is the number?
5. What is the power of 36 that gives $36^{1/2}$?
6. Point *X* is on each of two given intersecting lines. How many such points *X* are there?
7. Point *Y* is on each of two given circles. How many such points *Y*?
8. Point *Z* is on each of a given circle and a given ellipse. How many such *Z*?
9. Point *R* is on the coordinate plane so its distance from a given point *A* is less than 4. Show in a sketch where *R* could be.

Problems 10-11:

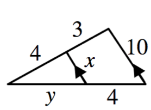


10. If the length of chord *AB* is *x* and length of *CB* is 16, what is *AC*?
11. If $AC = y$ and $CB = z$, how long is *AB* (in terms of *y* and *z*)?
12. This square is cut into two smaller squares and two non-square rectangles as shown.



Before being cut, the large square had area $(a+b)^2$. The two smaller squares have areas a^2 and b^2. Find the total area of the two non-square rectangles. Do the areas of the 4 parts add up to the area of the original square?

13. Find *x* and *y*:



14. When constructing an equilateral triangle with an area that is 100 times the area of a given equilateral triangle, what length should be used for a side?

Problems 15-16: *x* and *y* are numbers, and two *x*'s equal three *y*'s.

15. Which of *x* or *y* must be larger?
16. What is the ratio of *x* to *y*?

Problems 17-21: A plane has a certain speed in still air. In still air, it goes 1350 miles in 3 hours:

17. What is its (still air) speed?
18. How long does it take to fly 2000 miles?
19. How far does the plane go in *x* hours?
20. If the plane flies against a 50 mph headwind, what is its ground speed?
21. If it has fuel for 7.5 hours of flying time, how far can it go against this headwind?

Problems 22-32: Georgie and Porgie bake pies. Georgie can complete 30 pies an hour:

22. How many can he make in one minute?
23. How many can he make in 10 minutes?
24. How many can he make in *x* minutes?
25. How long does he take to make 200 pies?

Problems 26-28: Porgie can finish 45 pies an hour:

26. How many can she make in one minute?
27. How many can she make in 20 minutes?
28. How many can she make in *x* minutes?

Problems 29-32: If they work together, how many pies can they produce in:

29. 1 minute
30. *x* minutes
31. 80 minutes
32. 3 hours

Problems 33-41: A nurse needs to mix some alcohol solutions, given as a percent by weight of alcohol in water. Thus in a 3% solution, 3% of the weight would be alcohol. She mixes *x* grams of 3% solution, *y* grams of 10% solution, and 10 grams of pure water to get a total of 140 grams of a solution which is 8% alcohol:

33. In terms of x, how many grams of alcohol are in the 3% solution?
34. The y grams of 10% solution would include how many grams of alcohol?
35. How many grams of solution are in the final mix (the 8% solution)?
36. Write an expression in terms of x and y for the total number of grams in the 8% solution contributed by the three ingredients (the 3%, 10%, and water).
37. Use your last two answers to write a 'total grams equation'.
38. How many grams of alcohol are in the 8%?
39. Write an expression in terms of x and y for the total number of grams of alcohol in the final solution.
40. Use the last two answers to write a 'total grams of alcohol equation'.
41. How many grams of each solution are needed?

42. Half the square of a number is 18. What is the number?
43. If the square of twice a number is 81, what is the number?
44. Given a positive number x. The square of a positive number y is at least 4 times x. How small can y be?
45. Twice the square of half of a number is x. What is the number?

Problems 46-48: Half of x is the same as one-third of y:

46. Which of x and y is the larger?
47. Write the ratio $x : y$ as the ratio of two integers.
48. How many x's equal 30 y's?

Problems 49-50: A gathering has twice as many women as men. If W is the number of women and M is the number of men:

49. Which is correct: $2M=W$ or $M=2W$?
50. If there are 12 women, how many men are there?

Problems 51-53: If A is increased by 25%, it equals B:

51. Which is larger, B or the original A?
52. B is what percent of A?
53. A is what percent of B?

Problems 54-56: If C is decreased by 40%, it equals D:

54. Which is larger, D or the original C?
55. C is what percent of D?
56. D is what percent of C?

Problems 57-58: The length of a rectangle is increased by 25% and its width is decreased by 40%:

57. Its new area is what percent of its old area?
58. By what percent has the old area increased or decreased?

Problems 59-61: Your wage is increased by 20%, then the new amount is cut by 20% (of the new amount):

59. Will this result in a wage which is higher than, lower than, or the same as the original wage?
60. What percent of the original wage is this final wage?
61. If the above steps were reversed (20% cut followed by 20% increase), the final wage would be what percent of the original wage?

62. Find 3% of 36.
63. 55 is what percent of 88?
64. What percent of 55 is 88?
65. 45 is 3% of what number?
66. The 3200 people who vote in an election are 40% of the people registered to vote. How many are registered?
67. If you get 36 on a 40-question test, what percent is this?
68. What is the average of 87, 36, 48, 59, and 95?
69. If two test scores are 85 and 60, what minimum score on the next test would be needed for an overall average of 80?
70. The average height of 49 people is 68 inches. What is the new average height if a 78-inch person joins the group?

Problems 71-72: s varies directly as P, and $P = 56$ when $s = 14$:

71. Find s when $P = 144$.
72. Find P when $s = 144$.

Problems 73-74: A is proportional to r^2, and when $r = 10$, $A = 400\pi$.

73. Find A when $r = 15$.
74. Find r when $A = 36\pi$

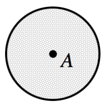
75. If b is inversely proportional to h, and $b = 36$ when $h = 12$, find h when $b = 3$.
76. If $3x = 4y$, write the ratio $x : y$ as the ratio of two integers.

77. The length of a rectangle is twice the width. If both dimensions are increased by 2 *cm*, the resulting rectangle has area 84 cm^2. What was the original width?

78. After a rectangular piece of knitted fabric shrinks in length 1 *cm* and stretches in width 2 *cm*, it is a square. If the original area was 40 cm^2, what is the square area?

Answers:

1. 144
2. $x+2,\ x-2$
3. 46
4. 9
5. $\frac{1}{2}$
6. 1
7. 0,1, or 2
8. 0, 1, 2, 3, or 4
9. Inside circle of radius 4 centered on A



10. $x-16$
11. $y+z$
12. 2*ab*, yes: $(a+b)^2 =$ $a^2+2ab+b^2$
13. $x=\frac{40}{7};\ \ y=\frac{16}{3}$
14. 10 times the original side
15. x
16. $\frac{3}{2}$
17. 450 mph
18. $4\frac{4}{9}$ hours
19. 450*x* miles
20. 400 mph
21. 3000 miles
22. $\frac{1}{2}$
23. 5
24. $\frac{x}{2}$
25. 400 min.
26. $\frac{3}{4}$
27. 15
28. $\frac{3x}{4}$
29. $\frac{5}{4}$
30. $\frac{5x}{4}$
31. 100
32. 225
33. .03*x*
34. .1*y*
35. 140
36. $x+y+10$
37. $x+y+10=140$
38. 11.2
39. $.03x+.1y$
40. $.03x+.1y=11.2$
41. $x=\frac{180}{7};\ \ y=\frac{730}{7}$
42. 6, -6
43. 4.5, -4.5
44. $2\sqrt{x}$
45. $\sqrt{2x}$
46. y
47. $3:2$ or $\frac{3}{2}$
48. 45
49. 2*M*=*W*
50. 6
51. *B*
52. 125%
53. 80%
54. *C*
55. $166\frac{2}{3}$%
56. 60%
57. 75%
58. 25% decrease
59. lower
60. 96%
61. same (96%)
62. 1.08
63. 62.5%
64. 160%
65. 1500
66. 8000
67. 90%
68. 65
69. 95
70. 68.2 inches
71. 36
72. 576
73. 900π
74. 3
75. 144
76. $4:3$
77. 5 *cm*
78. 49